# Appendix to 'Old, frail, and uninsured: Accounting for puzzles in the U.S. long-term care insurance market.'

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## 1 Proofs of propositions

For the proofs it is useful to denote $h(\cdot)$ as the density function associated with the distribution $H(\cdot)$ and define $\hat{\omega}(\pi,\iota) \equiv \underline{c}_{NH} + m - \iota + \pi$. Note that by Equation (10) in the paper, Medicaid transfers are zero for all $\omega \geq \hat{\omega}$ and positive for all $\omega < \hat{\omega}$.

**Proposition 1.** *If $\lambda > 1$ then the optimal menu features incomplete insurance for both types, i.e., $\iota^i < m$ for $i \in \{b, g\}$.*

*Proof.* The optimal menu must satisfy the first-order conditions, Equations (6) and (7) in the paper. First, note that the slope of the indifference curve at the full insurance level of indemnity always equals $\theta^i$ or

$$MRS(\theta^i, \pi^i, m) = \frac{\theta^i u'(\omega - \pi^i - m + \iota^i)}{\theta^i u'(\omega - \pi^i - m + \iota^i) + (1-\theta^i)u'(\omega - \pi^i)}\Big|_{\iota^i = m} = \theta^i, \tag{1}$$

for all $\pi^i$. Second, note that the slope of the indifference curve declines with the level of indemnity or

$$\frac{\partial MRS(\theta^i, \pi^i, \iota^i)}{\partial \iota^i} = \frac{\theta^i(1-\theta^i)u''(c_{NH})u'(c_o)}{[\theta^i u'(c_{NH}) + (1-\theta^i)u'(c_o)]^2} < 0. \tag{2}$$

The good type is always under-insured, regardless of whether the optimal contract is pooling or separating. To establish this result combine Equation (6) in the paper with Equation (1) to obtain the following inequality

$$MRS(\theta^g, \pi^g, \iota^g) = \lambda\eta > \theta^g = MRS(\theta^g, \pi^g, m),$$

which holds when $\lambda > 1$ since $\eta = \psi\theta^g + (1-\psi)\theta^b \geq \theta^g$. Then it follows from Equation (2) that $\iota^g < m$ and, in a pooling contract, $\iota^b = \iota^g < m$. If instead the equilibrium is separating then combine Equation (7) in the paper and Equation (1) to obtain

$$MRS(\theta^b, \pi^b, \iota^b) = \lambda\theta^b > \theta^b = MRS(\theta^b, \pi^b, m),$$

which holds when $\lambda > 1$. Thus, from Equation (2), $\iota^b < m$. □

**Proposition 2.** *There will be no trade, i.e., the optimal menu will consist of a single (0,0) contract iff*

$$MRS(\theta^b, 0, 0) \leq \lambda\theta^b, \tag{3}$$
$$MRS(\theta^g, 0, 0) \leq \lambda\eta, \tag{4}$$

*both hold.*

*Proof.* The proof follows directly from the proof of Proposition 2 in Chade and Schlee (2014) modified for the 2-type case. □

**Proposition 3.** *If $\underline{\omega} < \underline{c}_{NH}$ then the optimal menu features incomplete insurance for both types, i.e., $\iota^i < m$ for $i \in \{b, g\}$.*

*Proof.* We start by showing that if $\underline{\omega} < \underline{c}_{NH}$ then the optimal contract for any type $i \in \{b, g\}$, $(\pi^i, \iota^i)$, is such that $\hat{\omega}(\pi^i, \iota^i) > \underline{\omega}$. Suppose instead that $\hat{\omega}(\pi^i, \iota^i) \leq \underline{\omega}$. In this case, no one of type $i$ is on Medicaid in equilibrium. The utility function, Equation (12) in the paper, can be stated as

$$U(\theta^i, \pi^i, \iota^i) = \int_{\underline{\omega}}^{\overline{\omega}} \Big[\theta^i u(c_{NH}) + (1-\theta^i)u(c_o)\Big] dH(\omega),$$

where $c_{NH} = \omega - m + \iota^i - \pi^i$ and $c_o = \omega - \pi^i$, and the marginal rate of substitution is

$$MRS(\theta^i, \pi^i, \iota^i) = \frac{\theta^i \int_{\underline{\omega}}^{\overline{\omega}} u'(c_{NH}) dH(\omega)}{\theta^i \int_{\underline{\omega}}^{\overline{\omega}} u'(c_{NH}) dH(\omega) + (1-\theta^i) \int_{\underline{\omega}}^{\overline{\omega}} u'(c_o) dH(\omega)}.$$

Following the same proof strategy as that of Proposition 1 it is easy to show that if $\lambda \geq 1$ then $\iota^i \leq m$ for $i \in \{g, b\}$. However, since $\underline{\omega} < \underline{c}_{NH}$ we have

$$\underline{c}_{NH} + m - \iota^i + \pi^i \equiv \hat{\omega}(\pi^i, \iota^i) \leq \underline{\omega} < \underline{c}_{NH},$$

which implies that $\iota^i - \pi^i > m$ and since $\pi^i > 0$ it must be that $\iota^i > m$, a contradiction. We have established that the equilibrium contract for each type $i \in \{g, b\}$ must be such that $\hat{\omega}(\pi^i, \iota^i) > \underline{\omega}$.

If $\hat{\omega}(\pi^i, \iota^i) \geq \overline{\omega}$ then everyone of type $i$ is on Medicaid in equilibrium and the utility function, Equation (12) in the paper, can be stated as

$$U(\theta^i, \pi^i, \iota^i) = \int_{\underline{\omega}}^{\overline{\omega}} \Big[\theta^i u(\underline{c}_{NH}) + (1-\theta^i)u(c_o)\Big] dH(\omega),$$

where $c_o = \omega - \pi^i$. In this case, $MRS(\theta^i, \pi^i, \iota^i) = 0$ for all $(\pi^i, \iota^i)$ and the optimal contract is $(0, 0)$.

We now establish that for $i \in \{b, g\}$, $\iota^i < m$ holds when $\hat{\omega}(\pi^i, \iota^i) \in (\underline{\omega}, \overline{\omega})$ by showing that $\iota^i \geq m$ leads to a contradiction. The utility function, Equation (12) in the paper, can be stated as

$$U(\theta^i, \pi^i, \iota^i) = \int_{\underline{\omega}}^{\hat{\omega}(\pi^i,\iota^i)} \left[\theta^i u(\underline{c}_{NH}) + (1 - \theta^i) u(c_o)\right] dH(\omega) + \int_{\hat{\omega}(\pi^i,\iota^i)}^{\overline{\omega}} \left[\theta^i u(c_{NH}) + (1 - \theta^i) u(c_o)\right] dH(\omega),$$

where $c_{NH} = \omega - m + \iota^i - \pi^i$ and $c_o = \omega - \pi^i$, and the marginal rate of substitution is

$$\begin{aligned} MRS(\theta^i, \pi^i, \iota^i) &= \frac{\theta^i \int_{\hat{\omega}}^{\bar{\omega}} u'(c_{NH}) dH(\omega)}{\theta^i \int_{\hat{\omega}}^{\bar{\omega}} u'(c_{NH}) dH(\omega) + (1 - \theta^i) \int_{\underline{\omega}}^{\overline{\omega}} u'(c_o) dH(\omega)}, \\ &= \left[1 + \frac{(1 - \theta^i)}{\theta^i} \frac{\int_{\underline{\omega}}^{\overline{\omega}} u'(c_o) dH(\omega)}{\int_{\hat{\omega}}^{\overline{\omega}} u'(c_{NH}) dH(\omega)}\right]^{-1}. \end{aligned}$$

If $\iota^i \geq m$ then $MRS(\theta^i, \pi^i, \iota^i) < \theta^i$. To see this suppose that $MRS(\theta^i, \pi^i, \iota^i) \geq \theta^i$ which implies that

$$\begin{aligned} 1 + \frac{(1 - \theta^i)}{\theta^i} \frac{\int_{\underline{\omega}}^{\overline{\omega}} u'(c_o) dH(\omega)}{\int_{\hat{\omega}}^{\bar{\omega}} u'(c_{NH}) dH(\omega)} \leq \frac{1}{\theta^i} \Leftrightarrow \\ \int_{\underline{\omega}}^{\hat{\omega}} u'(c_o) dH(\omega) \leq \int_{\hat{\omega}}^{\bar{\omega}} \left[u'(c_{NH}) - u'(c_o)\right] dH(\omega). \end{aligned} \tag{5}$$

Since $\iota^i \geq m$, we have $c_{NH} = \omega - \pi^i + \iota^i - m \geq c_o = \omega - \pi^i$, which implies $u'(c_{NH}) - u'(c_o) \leq 0$. Equation (5) becomes

$$\int_{\underline{\omega}}^{\hat{\omega}} u'(c_o) dH(\omega) \leq \int_{\hat{\omega}}^{\bar{\omega}} \left[u'(c_{NH}) - u'(c_o)\right] dH(\omega) \leq 0, \tag{6}$$

which is a contradiction since $u'(c_0) > 0$ and $\underline{\omega} < \hat{\omega} < \overline{\omega}$.

Having established that if $\iota^i \geq m$, $MRS(\theta^i, \pi^i, \iota^i) < \theta^i$ the final step is to show that this condition violates the necessary conditions for an optimal contact for each type of agent. First, consider good types. Note that $\theta^g \leq \lambda\eta$ since $\lambda \geq 1$ and $\eta = \psi\theta^g + (1 - \psi)\theta^b > \theta^g$. So $MRS(\theta^g, \pi^g, \iota^g) < \lambda\eta$ when $\iota^g \geq m$. This is a contradiction because the equilibrium contract for good types must satisfy Equation (6) in the paper. It follows that $\iota^g < m$.

Second, consider bad types. In a pooling contract, $\iota^b = \iota^g < m$. If instead the equilibrium is separating then we have established that if $\iota^b \geq m$ then $MRS(\theta^b, \pi^b, \iota^b) < \theta^b \leq \lambda\theta^b$ since $\lambda \geq 1$. This is a contradiction because the equilibrium contract for bad types must satisfy Equation (7) in the paper. It follows that $\iota^b < m$.

□

**Lemma 1.** ***Single-crossing Property*** *The single-crossing property holds when the endowment is stochastic and Medicaid is present with $\underline{c}_{NH} > 0$.*

*Proof.* The proof shows that $\frac{\partial MRS(\theta,\pi,\iota)}{\partial\theta} > 0$ for all $\pi, \iota \in \mathbb{R}^+$. Recall that

$$MRS(\theta,\pi,\iota) = -\frac{U_\iota(\theta,\pi,\iota)}{U_\pi(\theta,\pi,\iota)},$$

where

$$U_\iota(\theta,\pi,\iota) = \theta \int_{\hat{\omega}}^{\bar{\omega}} u'(c_{NH})dH(\omega) > 0, \tag{7}$$
$$U_\pi(\theta,\pi,\iota) = -U_\iota(\theta,\pi,\iota) - (1-\theta)B < 0, \tag{8}$$

with $B \equiv \int_{\underline{\omega}}^{\overline{\omega}} u'(c_o)dH(\omega) > 0$. Differentiating the MRS with respect to $\theta$ yields

$$\frac{\partial MRS}{\partial \theta} = -\frac{U_{\iota\theta}U_\pi - \theta U_\iota U_{\pi\theta}}{U_\pi^2}, \tag{9}$$

where

$$U_{\iota\theta} = \theta^{-1}U_\iota > 0, \tag{10}$$
$$U_{\pi\theta} = -U_{\iota\theta} + B = -\theta^{-1}U_\iota + B, \tag{11}$$

and the arguments are omitted to save space. Using Equations (7)–(11) it is easy to show that

$$\frac{\partial MRS}{\partial \theta} = \frac{BU_{\iota\theta}}{\theta U_\pi^2} > 0.$$

□

**Proposition 4.** *If the optimal menu is a* $(0,0)$ *pooling contract then*

$$U(\theta^b, \lambda\theta^b\iota, \iota) < U(\theta^b, 0, 0), \quad \forall \iota \in \mathbb{R}_+, \tag{12}$$

*and*

$$U(\theta^g, \lambda\eta\iota, \iota) < U(\theta^g, 0, 0), \quad \forall \iota \in \mathbb{R}_+. \tag{13}$$

*Proof.* The proposition is proved by showing that if the conditions don't hold, one can find a menu with at least one nonzero contract that satisfies all the constraints and delivers non-negative profits.

First, assume that condition (12) does not hold but that condition (13) does. If (12) does not obtain, there exists $\iota \in \mathbb{R}_+$ such that

$$U(\theta^b, \lambda\theta^b\iota, \iota) \geq U(\theta^b, 0, 0). \tag{14}$$

Give bad types $(\lambda\theta^b\iota, \iota)$ and good types $(0, 0)$. Under this menu the insurer's profits are

$$\Pi = (1-\psi)\lambda\theta^b\iota + \psi 0 - (1-\psi)\lambda\theta^b\iota - \psi 0 = 0;$$

the participation constraint for the bad types, which is also their incentive compatibility constraint, holds by condition (14); the participation constraint of the good types is trivially satisfied; and the incentive compatibility constraint for the good types is satisfied since

$$U(\theta^g, \lambda\theta^b\iota, \iota) < U(\theta^g, \lambda\eta\iota, \iota) < U(\theta^g, 0, 0),$$

where the first inequality follows from the fact that $\eta < \theta^b$ and the second from condition (13).

Second, assume that condition (13) does not hold which means there exists $\iota \in \mathbb{R}^+$ such that

$$U(\theta^g, \lambda\eta\iota, \iota) \geq U(\theta^g, 0, 0). \tag{15}$$

Give both types $(\lambda\eta\iota, \iota)$. Under this pooling contract the insurer's profits are

$$\Pi = \lambda\eta\iota - \lambda\eta\iota = 0,$$

the participation constraint of the good types holds by condition (15), and the participation constraint for the bad types holds since condition (15) holds and $U$ satisfies the single-crossing property established in Lemma (1). Note that the incentive compatibility constraints are trivially satisfied since both types get the same contract.

□

**Proposition 5.** *When $\overline{\omega} - m \leq \underline{c}_{NH}$, the possibility of rejection in equilibrium increases if the distribution of endowments on $[\underline{\omega}, \overline{\omega}]$ is given by $H_1(\cdot)$ instead of $H(\cdot)$ where $H_1(\cdot)$ is first-order stochastically dominated by $H(\cdot)$.*

*Proof.* It is useful to express Equations (12)–(13) as

$$U(\theta^i, \pi^i, \iota) - U(\theta^i, 0, 0) < 0, \quad \forall \iota \in \mathbb{R}^+, i \in \{g, b\}, \tag{16}$$

where

$$U(\theta^i, \pi^i, \iota) = \int_{\underline{\omega}}^{\overline{\omega}} \left[\theta^i u(\max(\underline{c}_{NH}, \omega - \pi^i - m + \iota)) + (1-\theta^i)u(\omega - \pi^i)\right] dH(\omega),$$

with

$$\pi^i = \begin{cases} \lambda\theta^b\iota, & \text{if } i = b, \\ \lambda\eta\iota, & \text{if } i = g. \end{cases}$$

Without loss of generality, assume that $m \geq \iota \geq \pi > 0$.

Let $\Delta U(H)$ and $\Delta U(H_1)$ represent $U(\theta^i, \pi^i, \iota) - U(\theta^i, 0, 0)$ when the endowment distribution is given by $H(\cdot)$ and $H_1(\cdot)$, respectively. Then

$$\Delta U(H) = \int_{\underline{\omega}}^{\overline{\omega}} \tilde{u}(\omega) dH(\omega),$$

and

$$\Delta U(H_1) = \int_{\underline{\omega}}^{\overline{\omega}} \tilde{u}(\omega) dH_1(\omega),$$

where

$$\tilde{u}(\omega) = \Big[\theta^i u(\max(\underline{c}_{NH}, \omega - \pi^i - m + \iota)) + (1-\theta^i)u(\omega - \pi^i)\Big] - \\ \Big[\theta^i u(\max(\underline{c}_{NH}, \omega - m)) + (1-\theta^i)u(\omega)\Big].$$

If $\tilde{u}(\omega)$ is non-decreasing then $\Delta U(H) \geq \Delta U(H_1)$ and rejections are weakly more likely under $H_1$ than $H$. When $\overline{\omega} - m \leq \underline{c}_{NH}$ we have

$$\tilde{u}(\omega) = \Big[\theta^i u(\max(\underline{c}_{NH}, \omega - \pi^i - m + \iota)) + (1-\theta^i)u(\omega - \pi^i)\Big] - \Big[\theta^i u(\underline{c}_{NH}) + (1-\theta^i)u(\omega)\Big],$$

and

$$\frac{d\tilde{u}(\omega)}{d\omega} = \begin{cases} \theta^i u'(\omega - \pi^i - m + \iota) + (1-\theta^i)[u'(\omega - \pi) - u'(\omega)], & \omega - \pi^i + \iota > \underline{c}_{NH}, \\ (1-\theta^i)[u'(\omega - \pi^i) - u'(\omega)], & \omega - \pi^i + \iota \leq \underline{c}_{NH}. \end{cases}$$

It is easy to see that $\frac{d\tilde{u}(\omega)}{d\omega} > 0$.

□

Note that the assumption that $\overline{\omega} - m \leq \underline{c}_{NH}$ is sufficient but not necessary. Thus it may be possible to relax this assumption.

**Proposition 6.** *When $\lambda > 1$ and $\theta^b$ is sufficiently close to 1, the possibility of rejection in equilibrium increases if:*

1. *$\theta^b$ increases;*
2. *$\theta^b$ increases and $\theta^g$ decreases such that the mean NH entry probability $\eta \equiv \psi\theta^g + (1-\psi)\theta^b$ does not change.*

*Proof.* Note that the proof is for the no Medicaid case, although, as the quantitative results illustrate, the proposition holds even when Medicaid is present. Without Medicaid, rejection will occur in equilibrium iff

$$f^b(\theta^b) \equiv \lambda\theta^b - MRS(\theta^b, 0, 0) \geq 0, \tag{17}$$

and

$$f^g(\theta^g, \theta^b) \equiv \lambda\eta - MRS(\theta^g, 0, 0) \geq 0, \tag{18}$$

where $\eta \equiv \psi\theta^g + (1-\psi)\theta^b$.

1. Differentiating $f^b$ with respect to $\theta^b$ yields

$$\frac{f^b(\theta^b)}{d\theta^b} = \lambda - \frac{\int_{\underline{\omega}}^{\overline{\omega}} u'(\omega - m)dH(\omega) \int_{\underline{\omega}}^{\overline{\omega}} u(\omega)dH(\omega)}{[\theta^b \int_{\underline{\omega}}^{\overline{\omega}} u'(\omega - m)dH(\omega) + (1-\theta^b)\int_{\underline{\omega}}^{\overline{\omega}} u'(\omega)dH(\omega)]^2}. \tag{19}$$

When $\theta^b = 1$, Equation (19) is positive since

$$\frac{f^b(\theta^b)}{d\theta^b}|_{\theta^b=1} = \lambda - \frac{\int_{\underline{\omega}}^{\overline{\omega}} u'(\omega)dH(\omega)}{\int_{\underline{\omega}}^{\overline{\omega}} u'(\omega - m)dH(\omega)},$$

$\lambda \geq 1$ and $u'(\omega) < u'(\omega - m)$ for $m > 0$. It is easy to see that Equation (19) is increasing in $\theta^b$. Thus if $\theta^b$ is sufficiently close to 1, increasing $\theta^b$ will increase $f^b$. Differentiating $f^g$ with respect to $\theta^b$ yields

$$\frac{f^g(\theta^g, \theta^b)}{d\theta^b} = \lambda(1 - \psi) > 0.$$

Thus increasing $\theta^b$ increases $f^g$.

2. The proof that $f^b$ is increases is the same as in 1 since $f^b$ does not depend on $\theta^g$. The first term of $f^g$ does not change. The second term only depends $\theta^g$ and differentiating it with respect to $\theta^g$ yields

$$\frac{dMRS(\theta^g, 0, 0)}{d\theta^g} = \frac{\int_{\underline{\omega}}^{\overline{\omega}} u'(\omega - m)dH(\omega) \int_{\underline{\omega}}^{\overline{\omega}} u(\omega)dH(\omega)}{[\theta^g \int_{\underline{\omega}}^{\overline{\omega}} u'(\omega - m)dH(\omega) + (1 - \theta^g) \int_{\underline{\omega}}^{\overline{\omega}} u'(\omega)dH(\omega)]^2} > 0.$$

So $f^g$ increases when $\theta^g$ declines.

□

**Corollary 7.** *Consider two groups of individuals, 1 and 2, such that NH risk is on average higher in group 1 than group 2 ($\eta_1 > \eta_2$). If $\lambda > 1$ then it is possible that, in equilibrium, group 1 is rejected and group 2 is not ($\iota_1^i = 0 < \iota_2^i$, $i \in \{b, g\}$).*

*Proof.* Suppose group 2 is not rejected and group 1 has a higher $\theta^b$ then group 2 but is otherwise identical. Assume also that $\theta^b$ of group 2 is sufficiently close to 1 so that Proposition 6 holds. Then by part 1. of Proposition 6 it is possible that group 1 is rejected.

□

Another way to get LTCI take-up rates to be declining in $\eta$ is to assume that $\lambda = 1$ and instead suppose that $\underline{c}_{NH} > 0$ and that expected endowments are negatively correlated with NH entry rates. In particular, if one considers two risk groups $\eta_1 > \eta_2$ the same result can also obtain if the expected endowment of risk group 1 is lower than the expected endowment of risk group 2.

# 2 Details of the data work

Our HRS sample is constructed from the 1992 to 2012 waves of HRS and AHEAD. The sample is essentially the same as Braun et al. (2015) and Kopecky and Koreshkova (2014). Beyond adding additional data from 1992, 1994, and 2012, there are a few other changes. There is no censoring at -500 and 500 for asset values near 0. We assign an individual's 1998 weight (or post-1998 mean weight, if their 1998 weight is 0) to pre-1998 waves where their weight is 0. The main definitional novelties/changes are now provided. An individual is retired if his labor earnings are less than \$1500 (in 2000 dollars). An individual is considered to have ever had long-term care insurance if they report having been covered in half or more of their observed waves.

**Nursing home event** A nursing home event occurs when an individual spends 100 days or more in a nursing home within the approximately two year span between HRS interviews or within the period between their last interview and death. If the individual dies less than 100 days after their last interview, but at the time of their death had been in a nursing home for over 100 days, this also counts as a nursing home event. In the HRS, there are several (sometimes inconsistent) variables that provide information about the number of days spent in a nursing home. From the RAND dataset, we use the total nursing nights over all stays during the wave, as well as the number of days one has been in a nursing home (conditional on being in a nursing home at the time of the interview). This information is also pulled from the exit data, as well as the date of entry to a nursing home, provided the individual died there. Interview and death dates are used when a respondent reports having been continuously in a nursing home since the previous wave. Since the information is sometimes conflicting, and one piece often missing when another observed, a nursing home event is assigned if any of the variables suggest a person met the criteria described above.

**Permanent Income** To calculate permanent income, first sum the household heads social security and pension income and average this over all waves in which the household head is retired. The cumulative distribution of this average is defined as the permanent income, which ranges from 0 to 1. For singles, the household head is the respondent, and for couples it is the male.

**Wealth** We use the wealth variable ATOTA which is the sum of the value of owned real estate (including primary residence), vehicles, businesses, IRS/Keogh accounts, stocks, bonds, checking/savings accounts, CDs, treasury bills and "other savings and assets" less any debt reported.

## 2.1 Construction of the Frailty Index

Table 1 lists the variables we used to construct the frailty index for HRS respondents. The choice of these variables is based on Genworth and Mutual of Omaha LTCI underwriting guidelines. To construct the frailty index, first sum the variables listed in the first column of Table 1, assigning each a value according to the second column. Then divide this sum by the total number of variables observed for the individual in the year, as long as the total includes 30 or more variables. The construction of this frailty index mostly follows the guidelines laid out in Searle et al. (2008), and uses a set of HRS variables similar to the index created in Yang and Lee (2009). There are a couple of differences however. Primarily, a few variables that do not necessarily increase with age (e.g. drinking $>15$ drinks per week and smoking) were included. Also, cognitive tests are broken into parts which each count as separate variables, essentially increasing their weight in the index relative to Searle et al. (2008), which uses only a single variable for cognitive impairment. Nevertheless, our frailty distribution still closely resembles those of frailty indices used in other papers.

## 2.2 Evidence of private information

Hendren (2013) finds that self-assessed NH entry risk is only predictive of a NH event for individuals who would likely be rejected by insurers. Hendren's measure of a NH event is independent of the length of stay. Since we focus on stays that are at least 100 days, we repeat the logit analysis of Hendren (2013) using our definition of a NH stay and our HRS sample. We get qualitatively similar results. We restrict the sample to individuals ages 65–80. We find evidence of private information at the 10 year horizon (but not at the 6 year) in a subsample of this sample consisting of individuals who would likely be rejected by insurers. This sample includes individuals who have any ADL/IADL restriction, past stroke, or past nursing or home care. The p-value for a Wald test which restricts the coefficients on subjective probabilities to zero is 0.003 at the 10 year horizon and 0.169 at the 6 year. If all individuals above age 80 are included in the reject sample as well the p-values at both horizons are less than 0.000. For a sample of individuals who would likely not be rejected we are unable to find evidence of private information. The p-value for a Wald test which restricts the coefficients on subjective probabilities to zero is 0.210 at the 10 year horizon and 0.172 at the 6 year.

## 2.3 Description of the auxiliary simulation model

To obtain survival and lifetime NH entry probabilities by frailty and PE quintile groups, we use an auxiliary simulation model similar to that in Hurd et al. (2013). First, using a multinomial logit, we estimate transition probabilities between four states that we observe in the HRS: alive and dead, each with and without a nursing home event in the last two years. These probabilities depend on age, PE, NH event status, and frailty, including polynomials and interactions of these variables. Specifically, age is modeled as a cubic function, frailty as a quadratic, and the others are both linear. Interactions include age with each of the other first-order terms, as well as frailty with PE. We also simulate lifetime frailty paths because we need them since, in contrast to Hurd et al. (2013), we include frailty in the multinomial logit. This is done using estimates from a fixed effects regression of frailty on lagged frailty, age, and age squared.

Simulations begin at age 67. To get the initial distribution of explanatory variables, we first average frailty and population weights across all observations at which an individual is between ages 62-72. PE and the estimated fixed effect are constant within individuals. The initial distribution then draws 500,000 times from this person-level weighted distribution. The model simulates two-year transitions, following the structure of the HRS data, and assigns age of death by randomly choosing an age between their last living wave and the

[1]Note that a half year is added to death age to account for the fact that reported ages are the floor of a respondent's continuous age. Nursing home entry ages are similarly assigned, but we add only 0.2 years due to the 100 day requirement of a nursing home event. They are also upwardly bound by death age when both occur in the same wave.

Table 1: Health Variables for Frailty Index Construction

| Variable | Value |
|---|---|
| Some difficulty with ADL/IADLs: | |
| Eating | Yes=1, No=0 |
| Dressing | Yes=1, No=0 |
| Getting in/out of bed | Yes=1, No=0 |
| Using the toilet | Yes=1, No=0 |
| Bathing/shower | Yes=1, No=0 |
| Walking across room | Yes=1, No=0 |
| Walking several blocks | Yes=1, No=0 |
| Using the telephone | Yes=1, No=0 |
| Managing money | Yes=1, No=0 |
| Shopping for groceries | Yes=1, No=0 |
| Preparing meals | Yes=1, No=0 |
| Getting up from chair | Yes=1, No=0 |
| Stooping/kneeling/crouching | Yes=1, No=0 |
| Lift/carry 10 lbs | Yes=1, No=0 |
| Using a map | Yes=1, No=0 |
| Taking medications | Yes=1, No=0 |
| Climbing 1 flight of stairs | Yes=1, No=0 |
| Picking up a dime | Yes=1, No=0 |
| Reaching/ extending arms up | Yes=1, No=0 |
| Pushing/pulling large objects | Yes=1, No=0 |
| Cognitive Impairment: | |
| Immediate Word Recall | +.1 for each word not recalled (10 total)* |
| Delayed Word Recall | +.1 for each word not recalled (10 total)* |
| Serial 7 Test | +.2 for each incorrect substraction (5 total) |
| Backwards Counting | Failed test=1, 2nd attempt = .5, 1st attempt = 0 |
| Identifying obejcts & Pres/VP | .25 for each incorrect answer (4 total) |
| Identifying date | .25 for each incorrect answer (4 total) |
| Ever had one of following conditions: | |
| High Blood Pressure | Yes=1, No=0 |
| Diabetes | Yes=1, No=0 |
| Cancer | Yes=1, No=0 |
| Lung disease | Yes=1, No=0 |
| Heart disease | Yes=1, No=0 |
| Stroke | Yes=1, No=0 |
| Psychological problems | Yes=1, No=0 |
| Arthritis | Yes=1, No=0 |
| BMI $\geq 30$ | Yes=1, No=0 |
| Drinks 15+ alcoholic drinks per week | Yes=1, No=0 |
| Smokes Now | Yes=1, No=0 |
| Has smoked ever | Yes=1, No=0 |

*For the 1994 HRS cohort, 40 questions were asked (instead of 20) for word recall. In this year, each missed question receives weight 0.05.

death wave.[1]

# 3 Computation

Computing an equilibrium in our model is subtle because Medicaid NH benefits are means-tested and Medicaid is a secondary payer of NH benefits. Individual saving policies exhibit jumps and the demand for private insurance interacts in subtle ways with $q(\kappa)$, the distribution of consumption demand shocks.

We start by discretizing the endowment and frailty distributions. The number of grid points for endowments $\mathbf{w}$ is $ny = 101$ and frailty takes on $nf = 5$ grid points. The consumption demand shock $\kappa$ is also discretized: $nk = 50$.

The specific algorithm for computing an equilibrium proceeds as follows. First, we guess values for profits (which gives us dividends) and taxes and then we iterate over profits and taxes until profits converge and taxes satisfy the government budget constraint. In each iteration, we have to solve for allocations, contracts, and profits for each combination of endowments and frailty in the discretized state space. For each point in the discertized state space $(\mathbf{w}_s, f_j)$, $s = 1, nw$ and $j = 1, nf$, we guess a level of savings: $\hat{a}_{f_j,\mathbf{w}_s}$. Given $\hat{a}_{f_j,\mathbf{w}_s}$, we then solve for the optimal contracts as follows. The optimal contract for a risk group depends on which individuals of observable type $(\mathbf{w}_s, f_j)$ qualify for Medicaid if they incur the NH shock. Thus, it depends on the specific combinations of the $\kappa$ shock and the private type $i \in \{g, b\}$ that imply that an individual qualifies for Medicaid. Because of the non-convexities introduced by Medicaid the Kuhn-Tucker conditions of the insurer's problem are not sufficient. However, if one first assumes a distribution of individuals across Medicaid, then a contract satisfying the Kuhn-Tucker conditions is sufficient. So we solve for the optimal contract for all feasible combinations of individuals with different $\kappa$'s and $i$'s receiving Medicaid. The number of cases that has to be considered is large but it can reduced by the noting that for a given value of $\kappa$ if a bad type is on Medicaid the good type is also on Medicaid by the single-crossing property and that if a type $i$ qualifies for Medicaid for a value of $\kappa$ he will also qualify for Medicaid for all larger values of $\kappa$.

To solve for the optimal contracts for each Medicaid distribution, first we solve for the optimal pooling contract. Second, we check to see if an optimal separating menu exists. The contract of type $g$ under the optimal separating menu is the same as the optimal pooling contract. So we fix the good type's contract at the optimal pooling one and solve for the optimal separating contract of the bad type (if it exists).[2] The optimal contract for observable type $(\mathbf{w}_s, f_j)$ under the current guess for savings, $\hat{a}_{f_j,\mathbf{w}_s}$ is then the one that maximizes the insurer's profits. Finally, we iterate over savings until we find the value of savings that maximizes expected lifetime utility.

# 4 Additional Calibration Details

Table 2 lists many of the model's parameters and their calibrated values. The survival probabilities of each frailty and PE quintile in the quantitative model are shown in Figure

[2] If a separating menu doesn't exists it means one of the inequality constraints will be violated.

Table 2: Model Parameters

| Description | Parameter | Value |
|---|---|---|
| Risk aversion coefficient | $\sigma$ | 2 |
| Preference discount factor | $\beta$ | 0.94 |
| Retirement preference discount factor | $\alpha$ | 0.21 |
| Interest rate (annualized) | r | 0.0062 |
| Frailty distribution | $h$ | BETA(1.53,6.70) |
| Endowment distribution | $[w_y, w_o]'$ | LN(-0.32,0.80) |
| Copula parameter | $\rho_{f,w_y}$ | -0.4 |
| Demand shock distribution | $\kappa$ | $1-\kappa \sim$ LN(-1.08,0.245) |
| Fraction of good types | $\psi$ | 0.7 |
| Nursing home cost | m | 0.0956 |
| Insurer's variable cost of paying claims | $\lambda$ | 1.195 |
| Insurer's fixed cost of paying claims | $k$ | 0.016 |
| Medicaid consumption floor | $\underline{c}_{NH}$ | 0.02 |
| Welfare consumption floor | $\underline{c}_{o}$ | 0.02 |
| Tax rate | $\tau$ | 0.011 |

1.

# 5 Additional Results

Figures 2 –3 show how Medicaid and adverse selection influence pricing and coverage at alternative frailty and wealth levels. In the Baseline economy, coverage is roughly flat across frailty quintiles for the bad risk types. Removing private information reverses the pattern of coverage and reduces the variation in loads. Premia fall and coverage increases for good types across all frailty quintiles. Bad risks, in contrast, face higher loads and lower levels of coverage in the Full Information economy and see their coverage fall with frailty.[3]

Reducing the scale of Medicaid, in contrast, increases the level of coverage for both private information types at each frailty quintile. However, the loads are also higher. This effect is most pronounced at frailty Q5. In the baseline LTCI insurance covers 49% of the costs for the good risk type and 74% for the bad risk type. When Medicaid is scaled back, coverage increases to 68% for good risks and to 85% for bad risks. Notice also that coverage increases monotonically in frailty in the No Medicaid economy.

Figure 3 reports how coverage and loads vary by wealth quintile for the same three scenarios. In the Baseline and Full Information economies, as wealth increases, bad risk types experience lower coverage and higher loads. Good risk types experience somewhat higher coverage and lower loads as wealth is increased but the overall pattern is hump-shaped. Reducing the Medicaid NH benefit floor has a very big impact on the poor. Their demand for LTCI is inelastic and, as a result, they now face the highest loads but also receive

[3]This final result dates back to Arrow (1963) who demonstrates that the amount of insurance available to those with high risk exposures falls if insurance markets open after their risk exposure is observed.

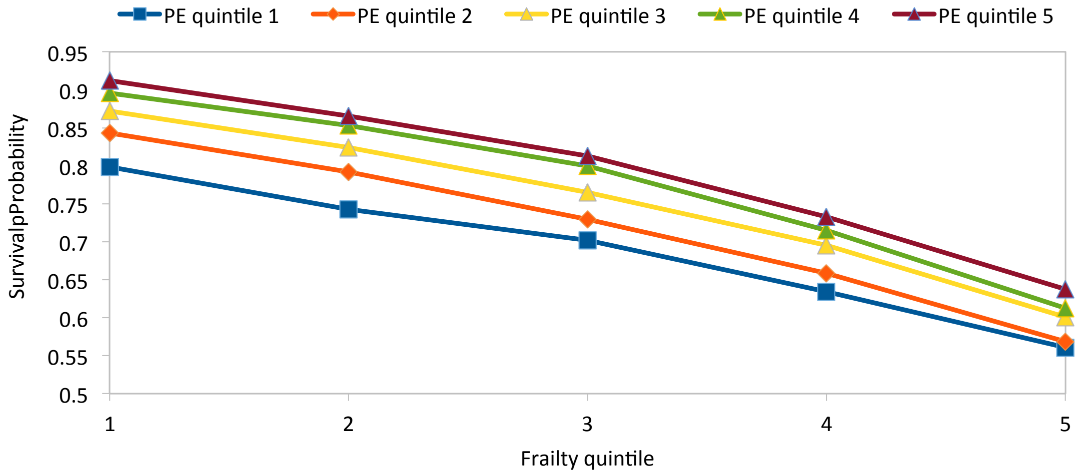


Figure 1: The probability of surviving to age 80 or until experiencing a NH stay by frailty and PE quintile. The probabilities are based on our auxiliary simulation model which is estimated using HRS data.

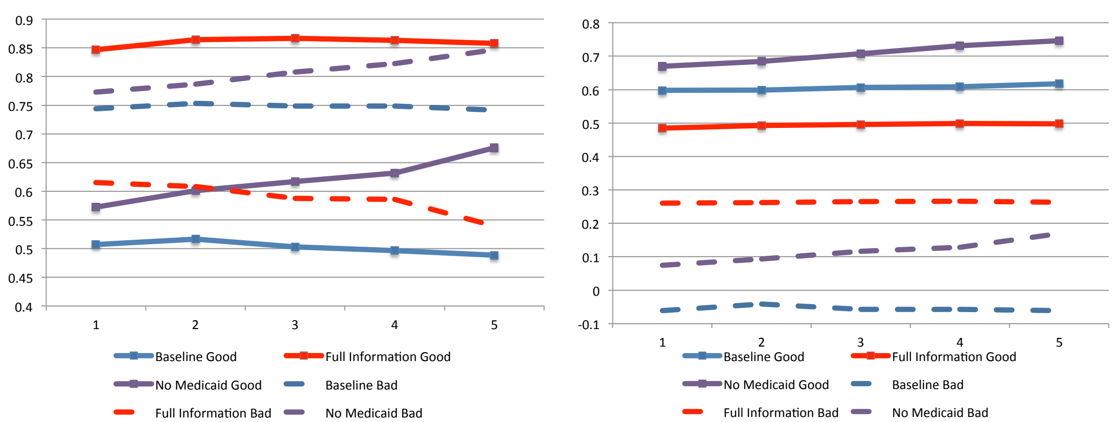


Figure 2: Insurance coverage and loads by frailty quintile.
The left panel reports LTCI indemnities relative to medical costs of a NH stay and the right panel reports loads for the two private information types: good risks and bad risks. Three economies are reported: Baseline, Full Information and No Medicaid.

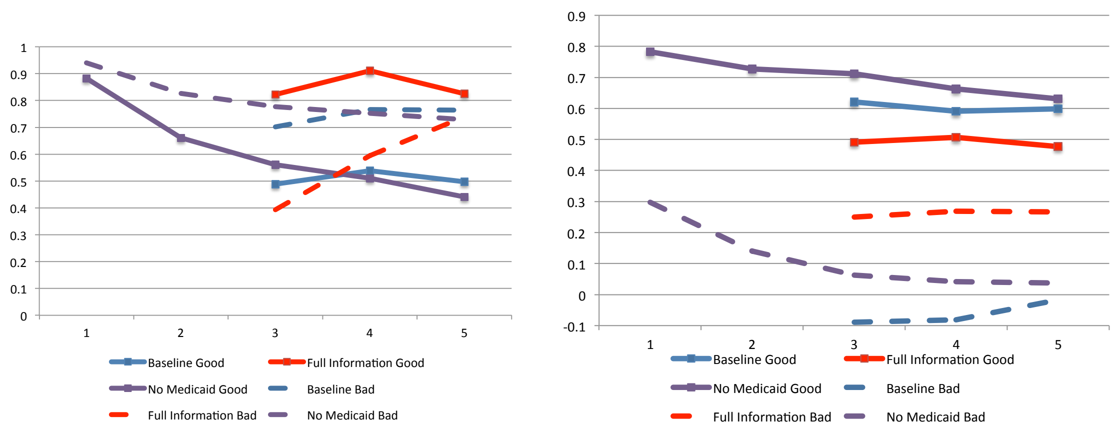


Figure 3: Insurance coverage and loads by wealth quintile.
The left panel reports indemnities relative to the medical cost of a NH stay and the right panel reports loads for the two private information types: good risks and bad risks. Three economies are reported: Baseline, Full Information and No Medicaid.

the most coverage.

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