Online Appendix to 'Old, frail, and uninsured: Accounting for features of the U.S. long-term care insurance market.'

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1 Additional theoretical results and discussion

1.1 Additional material for Section 3.1 of the paper

The insurer's problem in the one-period model discussed in Section 3.1 of the paper (administrative costs but no Medicaid) is

$$\max_{\pi^g,\iota^g,\pi^b,\iota^b} \psi\Big\{\pi^g - \theta^g\big[\lambda\iota^g + kI(\iota^g > 0)\big]\Big\} + (1-\psi)\Big\{\pi^b - \theta^b\big[\lambda\iota^b + kI(\iota^b > 0)\big]\Big\}, \tag{1}$$

subject to

$$(PC_i) \quad U(\theta^i, \pi^i, \iota^i) - U(\theta^i, 0, 0) \geq 0, \quad i \in \{g, b\}, \tag{2}$$

$$(IC_i) \quad U(\theta^i, \pi^i, \iota^i) - U(\theta^i, \pi^j, \iota^j) \geq 0, \quad i, j \in \{g, b\},\ i \neq j, \tag{3}$$

where $\lambda \geq 1$ captures variable costs and $k \geq 0$ captures fixed costs.

Denote consumption of an individual with risk type i as c^i_{NH} in the NH state and c^i_o otherwise. An individual's utility function is

$$\begin{aligned} U(\theta^i, \pi^i, \iota^i) =& \theta^i u(\omega - \pi^i - m + \iota^i) + (1-\theta^i)u(\omega - \pi^i), \\ =& \theta^i u(c^i_{NH}) + (1-\theta^i)u(c^i_o), \end{aligned} \tag{4}$$

and the associated marginal rate of substitution between premium and indemnity is

$$\frac{\partial \pi}{\partial \iota}(\theta^i) = -\frac{U_\iota(\cdot)}{U_\pi(\cdot)} = \frac{\theta^i u'(c^i_{NH})}{\theta^i u'(c^i_{NH}) + (1-\theta^i)u'(c^i_o)} \equiv MRS(\theta^i, \pi^i, \iota^i). \tag{5}$$

Assume that the utility function has the property that $MRS(\theta^i, \pi^i, \iota^i)$ is strictly increasing in $\theta^i, i \in \{g, b\}$. This is true, for example, if $u'(\cdot) > 0$. Under this assumption, which is referred to as the single crossing property, any menu of contracts that satisfies incentive compatibility will have the property that if $\theta^{i'} > \theta^i$ then $\pi^{i'} \geq \pi^i$ and $\iota^{i'} \geq \iota^i$.

At the optimal menu, Equation (2) will bind for the good types and Equation (3) will bind for the bad types. If the optimal menu features positive insurance than it will also satisfy the two first-order conditions

$$\psi MRS(\theta^g, \pi^g, \iota^g) + (1-\psi)\left[\frac{U_\pi(\theta^b, \pi^g, \iota^g)}{U_\pi(\theta^b, \pi^b, \iota^b)} MRS(\theta^g, \pi^g, \iota^g) + \frac{U_\iota(\theta^b, \pi^g, \iota^g)}{U_\pi(\theta^b, \pi^b, \iota^b)}\right] = \lambda\psi\theta^g, \quad (6)$$

$$MRS(\theta^b, \pi^b, \iota^b) = \lambda\theta^b. \quad (7)$$

When $\lambda = 1$ and $k = 0$, the equilibrium menu is standard. This means that it will always be a separating one with bad types receiving full insurance. When $\lambda > 1$ and/or $k > 0$, the optimal menu may be a pooling menu. In the case of $k > 0$ and $\lambda = 1$ the only type of pooling menu that can arise is a no-trade menu. When $\lambda > 1$ both no-trade and pooling menus featuring positive insurance can arise. An optimal pooling menu featuring positive insurance must satisfy

$$MRS(\theta^g, \pi, \iota) = \lambda\eta, \quad (8)$$

$$U(\theta^g, \pi, \iota) - U(\theta^g, 0, 0) = 0. \quad (9)$$

These two equations can be derived using from Equations (2), (3), (6) and (7), where $\pi = \pi^g = \pi^b$ and $\iota = \iota^b = \iota^b$.

For simplicity, the good types' contract in Figures 1a and 1b in the paper are illustrated as the optimal pooling contract. Rearranging the first-order conditions, one can show that Equation (6) is equivalent to

$$MRS(\theta^g, \pi^g, \iota^g) = \lambda\left[\frac{\psi\theta^g + (1-\psi)\theta^b A}{\psi + (1-\psi)B}\right], \quad (10)$$

where $A \equiv U_\iota(\theta^b, \pi^g, \iota^g)/U_\iota(\theta^b, \pi^b, \iota^b)$ and $B \equiv U_\pi(\theta^b, \pi^g, \iota^g)/U_\pi(\theta^b, \pi^b, \iota^b)$. The figure corresponds to cases where A and B are approximately 1.

Proposition 1. *If $\lambda > 1$ then the optimal menu features incomplete insurance for both types, i.e., $\iota^i < m$ for $i \in \{b, g\}$.*

Proof. First, note that the slope of the indifference curve at the full insurance level of indemnity always equals θ^i or

$$MRS(\theta^i, \pi^i, m) = \frac{\theta^i u'(\omega - \pi^i - m + \iota^i)}{\theta^i u'(\omega - \pi^i - m + \iota^i) + (1-\theta^i)u'(\omega - \pi^i)}\Bigg|_{\iota^i = m} = \theta^i, \quad (11)$$

for all π^i. Second, note that the slope of the indifference curve declines with the level of indemnity or

$$\frac{\partial MRS(\theta^i, \pi^i, \iota^i)}{\partial \iota^i} = \frac{\theta^i(1-\theta^i)u''(c_{NH})u'(c_o)}{\left[\theta^i u'(c_{NH}) + (1-\theta^i)u'(c_o)\right]^2} < 0. \quad (12)$$

The good type is always under-insured, regardless of whether the optimal contract is pooling or separating. To see this for the optimal pooling contract (π^p, ι^p), combine Equation (8) with Equation (11) to obtain the following inequality

$$MRS(\theta^g, \pi^p, \iota^p) = \lambda\eta > \theta^g = MRS(\theta^g, \pi^p, m),$$

which holds when $\lambda > 1$ since $\eta = \psi\theta^g + (1-\psi)\theta^b \geq \theta^g$. Then it follows from Equation (12) that $\iota^p < m$. If instead the equilibrium is separating then combine the expression for Equation (10) and Equation (11) to obtain

$$MRS(\theta^g, \pi^g, \iota^g) = \lambda \left[\frac{\psi\theta^g + (1-\psi)\theta^b A}{\psi + (1-\psi)B} \right] > \theta^g = MRS(\theta^g, \pi^g, m),$$

where $A \equiv U_\iota(\theta^b, \pi^g, \iota^g)/U_\iota(\theta^b, \pi^b, \iota^b)$ and $B \equiv U_\pi(\theta^b, \pi^g, \iota^g)/U_\pi(\theta^b, \pi^b, \iota^b)$. The inequality holds for $\lambda \geq 1$ since under the single-crossing property any incentive compatible separating contract must be such that $\pi^g < \pi^b$ which implies that

$$A = \frac{\theta^b u'(c^g_{NH})}{\theta^b u'(c^b_{NH})} > \frac{\theta^b u'(c^g_{NH}) + (1-\theta^b)u'(c^g_o)}{\theta^b u'(c^b_{NH}) + (1-\theta^b)u'(c^b_o)} = B,$$

where $c^i_{NH} = \omega - m + \iota^i - \pi^i$ and $c^i_o = \omega - \pi^i$. Thus, from Equation (12), $\iota^g < m$. Finally, combine Equation (7) and Equation (11) to obtain

$$MRS(\theta^b, \pi^b, \iota^b) = \lambda\theta^b > \theta^b = MRS(\theta^b, \pi^b, m),$$

which holds when $\lambda > 1$. So, from Equation (12), $\iota^b < m$. □

Proposition 2. *There will be no trade, i.e., the optimal menu will consist of a single* $(0,0)$ *contract iff*

$$MRS(\theta^b, 0, 0) \leq \lambda\theta^b, \tag{13}$$
$$MRS(\theta^g, 0, 0) \leq \lambda\eta, \tag{14}$$

both hold.

Proof. Assume that u is strictly concave so that $u'(\cdot) > 0$ and $u''(\cdot) < 0$ and $k = 0$. First, we will show that if Equations (13) and (14) hold then the optimal menu will be a single $(0,0)$ contract.

Part 1: We will show that if Equation (13) holds then the optimal menu must be a pooling menu. Suppose the optimal menu features a contract for the good types (π^g, ι^g) with $\iota^g \geq \pi^g \geq 0$ and a contract for the bad types (π^b, ι^b) with $\iota^b \geq \pi^b \geq 0$. The following inequalities hold:

$$\theta^b\lambda \geq MRS(\theta^b, 0, 0) \geq MRS(\theta^g, \iota^g, \pi^g). \tag{15}$$

The first inequality is Equation (13) and the second follows from $u'(\cdot) > 0$, $\iota^g \geq \pi^g \geq 0$, and $\theta^b > \theta^g$. By single-crossing we have that $\pi^b \geq \pi^g$ and $\iota^b \geq \iota^g$. This together with the fact that u is strictly concave means that

$$\pi^b - \pi^g \leq MRS(\theta^g, \iota^g, \pi^g)(\iota^b - \iota^g). \tag{16}$$

Combining (15) and (16) yields

$$\pi^b - \pi^g \leq \theta^b \lambda(\iota^b - \iota^g), \tag{17}$$

and rearranging gives

$$\pi^g - \theta^b \lambda \iota^g \geq \pi^b - \theta^b \lambda \iota^b). \tag{18}$$

However, this implies that giving the bad types (π^g, ι^g) which they value as equal to (π^b, ι^b) does not reduce (and may increase) profits. The only way this can be is if $(\pi^g, \iota^g) = (\pi^b, \iota^b) \equiv (\pi, \iota)$.

Part 2: We will show that if Equation (14) holds then the optimal pooling contract must be a no-trade, $(0, 0)$ contract. If $\pi > 0$ and $\iota > 0$ then the following inequalities hold:

$$\eta\lambda \geq MRS(\theta^g, 0, 0) > MRS(\theta^g, \iota, \pi). \tag{19}$$

The first inequality is Equation (14) and the second follows from $u'(\cdot) > 0$ and $\iota \geq \pi > 0$. Thus (π, ι) does not satisfy the first-order optimality conditions for a pooling contract. The pooling contract must be $(0, 0)$.

Now, we will show that if the optimal menu is a single $(0, 0)$ contract then Equations (13) and (14) hold. Suppose Equation (13) does not hold. Then there exists a menu that gives $(0, 0)$ to good types and a small amount of insurance to bad types with a contract (π^b, ι^b) that satisfies

$$MRS(\theta^b, \pi^b, \iota^b) \geq \lambda\theta^b,$$

and with a premium chosen such that $U(\theta^b, \pi^b, \iota^b) = U(\theta^b, 0, 0)$. This menu satisfies all the constraints of the insurer's problem and delivers higher profits to the insurer. Thus the optimal menu can not consistent of a single $(0, 0)$ contract.

Suppose Equation (14) does not hold. Then there exists a pooling contract (π, ι) that gives a small amount of insurance to both types and satisfies

$$MRS(\theta^g, \pi, \iota) \geq \lambda\eta,$$

with the premium such that $U(\theta^g, \pi, \iota) = U(\theta^g, 0, 0)$. This menu satisfies all the constraints of the insurer's problem and delivers higher profits to the insurer. Thus the optimal menu can not consistent of a single $(0, 0)$ contract.

$\square$

Intuition: No trade equilibria occur when the amount individuals are willing to pay for even a small positive separating or pooling equilibrium is less than the amount required to provide nonnegative profits to the insurer. Condition (13) rules out profitable separating menus where only bad types have positive insurance, such as the one illustrated in Figure 1e in the paper. Condition (14) rules out profitable pooling and separating menus where both types are offered positive insurance.

1.2 Additional material for Section 3.2 of the paper

The insurer's problem in the version of the one-period model discussed in Section 3.2 of the paper (Medicaid but no administrative costs) is

$$\max_{\pi^g,\iota^g,\pi^b,\iota^b} \psi\Big\{\pi^g - \theta^g\iota^g\Big\} + (1-\psi)\Big\{\pi^b - \theta^b\iota^b\Big\}, \tag{20}$$

subject to

$$\begin{aligned} (PC_i) \quad & U(\theta^i,\pi^i,\iota^i) - U(\theta^i,0,0) \geq 0, \quad i \in \{g,b\}, & (21)\\ (IC_i) \quad & U(\theta^i,\pi^i,\iota^i) - U(\theta^i,\pi^j,\iota^j) \geq 0, \quad i,j \in \{g,b\},\ i \neq j, & (22) \end{aligned}$$

where an individual's utility function is

$$U(\theta^i,\pi^i,\iota^i) = \int_{\underline{\omega}}^{\overline{\omega}} \Big[\theta^i u(c^i_{NH}(\omega)) + (1-\theta^i)u(c^i_o(\omega))\Big] dH(\omega), \tag{23}$$

with

$$\begin{aligned} c^i_o(\omega) &= \omega - \pi^i, & (24)\\ c^i_{NH}(\omega) &= \omega + TR(\omega,\pi^i,\iota^i) - \pi^i - m + \iota^i. & (25) \end{aligned}$$

The Medicaid transfer is defined by Equation (4) in the paper.

We first show that, if $u'(\cdot) > 0$, the single-crossing property continues to obtain when Medicaid is present and the endowment is stochastic.

Lemma 1. ***(Single-crossing Property)*** *If $u'(\cdot) > 0$, the single-crossing property holds when the endowment is stochastic and Medicaid is present with $\underline{c}_{NH} > 0$.*

Proof. Denote $h(\cdot)$ as the density function associated with the distribution $H(\cdot)$ and define $\hat{\omega}(\pi,\iota) \equiv \underline{c}_{NH} + m - \iota + \pi$. Note that by Equation (4) in the paper, Medicaid transfers are zero for all $\omega \geq \hat{\omega}$ and positive for all $\omega < \hat{\omega}$. The proof shows that $\frac{\partial MRS(\theta,\pi,\iota)}{\partial\theta} > 0$ for all $\pi,\iota \in \mathbb{R}^+$. Recall that

$$MRS(\theta,\pi,\iota) = -\frac{U_\iota(\theta,\pi,\iota)}{U_\pi(\theta,\pi,\iota)},$$

where

$$\begin{aligned} U_\iota(\theta,\pi,\iota) &= \theta\int_{\hat{\omega}}^{\bar{\omega}} u'(c_{NH})dH(\omega) > 0, & (26)\\ U_\pi(\theta,\pi,\iota) &= -U_\iota(\theta,\pi,\iota) - (1-\theta)B < 0, & (27) \end{aligned}$$

with $B \equiv \int_{\underline{\omega}}^{\overline{\omega}} u'(c_o)dH(\omega) > 0$. Differentiating the MRS with respect to θ yields

$$\frac{\partial MRS}{\partial\theta} = -\frac{U_{\iota\theta}U_\pi - \theta U_\iota U_{\pi\theta}}{U_\pi^2}, \tag{28}$$

where

$$U_{\iota\theta} = \theta^{-1} U_\iota > 0, \tag{29}$$
$$U_{\pi\theta} = -U_{\iota\theta} + B = -\theta^{-1} U_\iota + B, \tag{30}$$

and the arguments are omitted to save space. Using Equations (26)–(30) it is easy to show that

$$\frac{\partial MRS}{\partial \theta} = \frac{BU_{\iota\theta}}{\theta U_\pi^2} > 0.$$

□

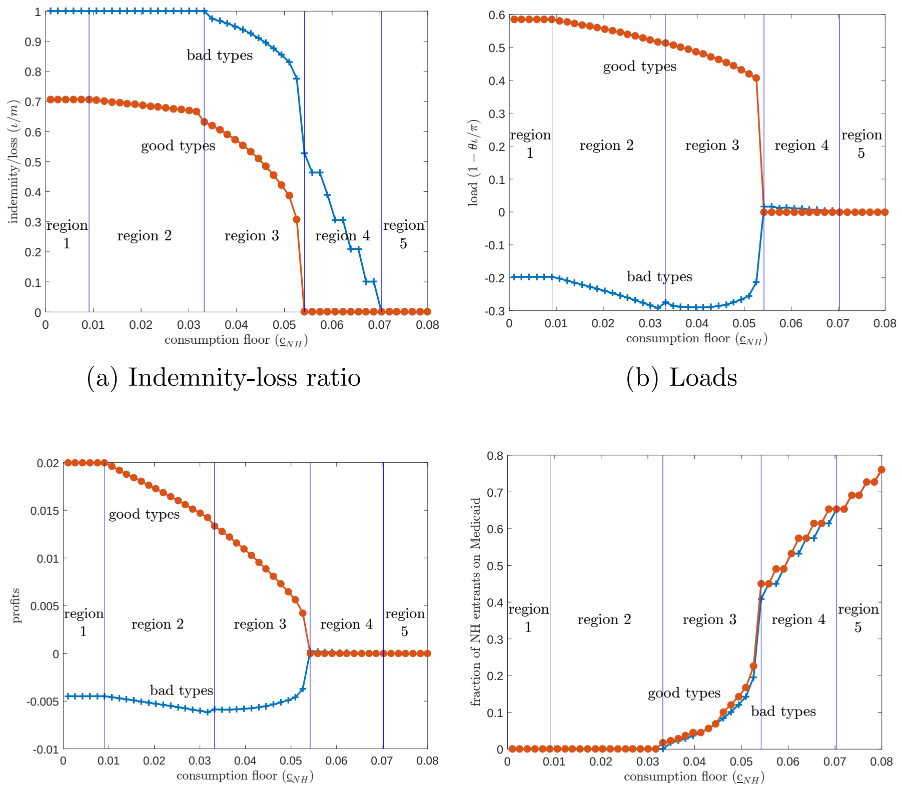
Figure 1 illustrates how the optimal contracts, profits and Medicaid take-up rates evolve as the Medicaid consumption floor, $\underline{c}_{NH}$, is increased from zero in the setup with endowment uncertainty. The figure is divided into 5 distinct regions. In region 1, the consumption floor is so low that even if an individual has no private LTCI and the smallest realization of the endowment he will not qualify for Medicaid. In this region, Medicaid has no effect on the optimal contracts. In region 2, Medicaid influences the contracts even though, in equilibrium, neither type receives Medicaid transfers. In this region, Medicaid has a similar effect to that illustrated in Figure 2b in the paper. For some realizations of the endowment, good types qualify for Medicaid if the contract is $(0, 0)$. This tightens their participation constraint and the contract offered to them has to be improved. A better contract for good types tightens, in turn, the incentive compatibility constraint for bad types. The insurer responds by reducing premiums for both types, and the indemnity of the good types and loads on both types fall. Since Medicaid's presence has resulted in more favorable contracts for individuals, the insurer's profits fall. In region 3, Medicaid has the same effects as in region 2 but now, in addition, both types receive Medicaid benefits in equilibrium for some realizations of ω. As discussed above, the partial insurance of NH shocks via Medicaid results in optimal contracts that feature partial coverage and, in this region, both types have less than full private insurance. Proposition 3 provides a sufficient condition for this to occur.

Proposition 3. *If $\underline{\omega} < \underline{c}_{NH}$ then the optimal menu features incomplete insurance for both types, i.e., $\iota^i < m$ for $i \in \{b, g\}$.*

Proof. Denote $h(\cdot)$ as the density function associated with the distribution $H(\cdot)$ and define $\hat{\omega}(\pi, \iota) \equiv \underline{c}_{NH} + m - \iota + \pi$. Note that by Equation (4) in the paper, Medicaid transfers are zero for all $\omega \geq \hat{\omega}$ and positive for all $\omega < \hat{\omega}$.

We start by showing that if $\underline{\omega} < \underline{c}_{NH}$ then the optimal contract for any type $i \in \{b, g\}$, (π^i, ι^i), is such that $\hat{\omega}(\pi^i, \iota^i) > \underline{\omega}$. Suppose instead that $\hat{\omega}(\pi^i, \iota^i) \leq \underline{\omega}$. In this case, no one of type i is on Medicaid in equilibrium. The utility function, Equation (6) in the paper, can be stated as

$$U(\theta^i, \pi^i, \iota^i) = \int_{\underline{\omega}}^{\overline{\omega}} \Big[\theta^i u(c_{NH}) + (1 - \theta^i) u(c_o)\Big] dH(\omega),$$



(a) Indemnity-loss ratio (b) Loads

(c) Profits (d) Fraction of NH entrants on Medicaid

Figure 1: Impact of varying the Medicaid consumption floor, $\underline{c}_{NH}$, on the indemnity-loss ratio, loads, profits, and the fraction of NH entrants on Medicaid when the endowment is stochastic.

where $c_{NH} = \omega - m + \iota^i - \pi^i$ and $c_o = \omega - \pi^i$, and the marginal rate of substitution is

$$MRS(\theta^i, \pi^i, \iota^i) = \frac{\theta^i \int_{\underline{\omega}}^{\overline{\omega}} u'(c_{NH}) dH(\omega)}{\theta^i \int_{\underline{\omega}}^{\overline{\omega}} u'(c_{NH}) dH(\omega) + (1-\theta^i) \int_{\underline{\omega}}^{\overline{\omega}} u'(c_o) dH(\omega)}.$$

Following the same proof strategy as that of Proposition 1 it is easy to show that if $\lambda \geq 1$ then $\iota^i \leq m$ for $i \in \{g, b\}$. However, since $\underline{\omega} < \underline{c}_{NH}$ we have

$$\underline{c}_{NH} + m - \iota^i + \pi^i \equiv \hat{\omega}(\pi^i, \iota^i) \leq \underline{\omega} < \underline{c}_{NH},$$

which implies that $\iota^i - \pi^i > m$ and since $\pi^i > 0$ it must be that $\iota^i > m$, a contradiction. We have established that the equilibrium contract for each type $i \in \{g, b\}$ must be such that $\hat{\omega}(\pi^i, \iota^i) > \underline{\omega}$.

If $\hat{\omega}(\pi^i, \iota^i) \geq \overline{\omega}$ then everyone of type i is on Medicaid in equilibrium and the utility function, Equation (6) in the paper, can be stated as

$$U(\theta^i, \pi^i, \iota^i) = \int_{\underline{\omega}}^{\overline{\omega}} \Big[\theta^i u(\underline{c}_{NH}) + (1-\theta^i) u(c_o)\Big] dH(\omega),$$

where $c_o = \omega - \pi^i$. In this case, $MRS(\theta^i, \pi^i, \iota^i) = 0$ for all (π^i, ι^i) and the optimal contract is $(0, 0)$.

We now establish that for $i \in \{b, g\}$, $\iota^i < m$ holds when $\hat{\omega}(\pi^i, \iota^i) \in (\underline{\omega}, \overline{\omega})$ by showing that $\iota^i \geq m$ leads to a contradiction. The utility function, Equation (6) in the paper, can be stated as

$$U(\theta^i, \pi^i, \iota^i) = \int_{\underline{\omega}}^{\hat{\omega}(\pi^i,\iota^i)} \Big[\theta^i u(\underline{c}_{NH}) + (1-\theta^i)u(c_o)\Big] dH(\omega) + \int_{\hat{\omega}(\pi^i,\iota^i)}^{\overline{\omega}} \Big[\theta^i u(c_{NH}) + (1-\theta^i)u(c_o)\Big] dH(\omega),$$

where $c_{NH} = \omega - m + \iota^i - \pi^i$ and $c_o = \omega - \pi^i$, and the marginal rate of substitution is

$$MRS(\theta^i, \pi^i, \iota^i) = \frac{\theta^i \int_{\hat{\omega}}^{\bar{\omega}} u'(c_{NH})dH(\omega)}{\theta^i \int_{\hat{\omega}}^{\bar{\omega}} u'(c_{NH})dH(\omega) + (1-\theta^i)\int_{\underline{\omega}}^{\overline{\omega}} u'(c_o)dH(\omega)}, = \left[1 + \frac{(1-\theta^i)}{\theta^i}\frac{\int_{\underline{\omega}}^{\overline{\omega}} u'(c_o)dH(\omega)}{\int_{\hat{\omega}}^{\bar{\omega}} u'(c_{NH})dH(\omega)}\right]^{-1}.$$

If $\iota^i \geq m$ then $MRS(\theta^i, \pi^i, \iota^i) < \theta^i$. To see this suppose that $MRS(\theta^i, \pi^i, \iota^i) \geq \theta^i$ which implies that

$$1 + \frac{(1-\theta^i)}{\theta^i}\frac{\int_{\underline{\omega}}^{\overline{\omega}} u'(c_o)dH(\omega)}{\int_{\hat{\omega}}^{\bar{\omega}} u'(c_{NH})dH(\omega)} \leq \frac{1}{\theta^i} \Leftrightarrow$$
$$\int_{\underline{\omega}}^{\hat{\omega}} u'(c_o)dH(\omega) \leq \int_{\hat{\omega}}^{\bar{\omega}} \left[u'(c_{NH}) - u'(c_o)\right] dH(\omega). \tag{31}$$

Since $\iota^i \geq m$, we have $c_{NH} = \omega - \pi^i + \iota^i - m \geq c_o = \omega - \pi^i$, which implies $u'(c_{NH}) - u'(c_o) \leq 0$. Equation (31) becomes

$$\int_{\underline{\omega}}^{\hat{\omega}} u'(c_o)dH(\omega) \leq \int_{\hat{\omega}}^{\bar{\omega}} \left[u'(c_{NH}) - u'(c_o)\right] dH(\omega) \leq 0, \tag{32}$$

which is a contradiction since $u'(c_0) > 0$ and $\underline{\omega} < \hat{\omega} < \overline{\omega}$.

Having established that $\iota^i \geq m$ implies $MRS(\theta^i, \pi^i, \iota^i) < \theta^i$ for $i \in \{b, g\}$ the final step is to show that this condition violates the necessary conditions for an optimal contact. First consider an optimal pooling contract (π^p, ι^p). Note that $\theta^g < \lambda\eta$ since $\lambda \geq 1$ and $\eta = \psi\theta^g + (1-\psi)\theta^b > \theta^g$. So $MRS(\theta^g, \pi^p, \iota^p) < \lambda\eta$ when $\iota^p \geq m$. This is a contradiction because the optimal pooling contract must satisfy Equation (8). It follows that $\iota^p < m$.

Now consider an optimal separating contract. First, consider good types. Under the optimal contract it must be that $\theta^g < \lambda(\psi\theta^g + (1-\psi)\theta^b A)/(\psi + (1-\psi)B)$, since $\lambda \geq 1$ and due to single-crossing (established in Lemma 1) and incentive compatibility $\pi^g < \pi^b$ so

$$A = \frac{\theta^b \int_{\hat{\omega}(\pi^g,\iota^g))}^{\bar{\omega}} u'(c^g_{NH})dH(\omega)}{\theta^b \int_{\hat{\omega}(\pi^b,\iota^b)}^{\bar{\omega}} u'(c^b_{NH})dH(\omega)} > \frac{\theta^b \int_{\hat{\omega}(\pi^g,\iota^g))}^{\bar{\omega}} u'(c^g_{NH})dH(\omega) + (1-\theta^b)\int_{\underline{\omega}}^{\bar{\omega}} u'(c^g_o)dH(\omega)}{\theta^b \int_{\hat{\omega}(\pi^b,\iota^b)}^{\bar{\omega}} u'(c^b_{NH})dH(\omega) + (1-\theta^b)\int_{\underline{\omega}}^{\bar{\omega}} u'(c^b_o)dH(\omega)} = B,$$

where $c_{NH}^i = \omega - m + \iota^i - \pi^i$ and $c_o^i = \omega - \pi^i$. Hence $MRS(\theta^g, \pi^g, \iota^g) < \lambda(\psi\theta^g + (1-\psi)\theta^b A)/(\psi + (1-\psi)B)$ when $\iota^g \geq m$. This is a contradiction because the equilibrium contract for good types must satisfy Equation (6). It follows that $\iota^g < m$.

Second, consider bad types. We have established that if $\iota^b \geq m$ then $MRS(\theta^b, \pi^b, \iota^b) < \theta^b \leq \lambda\theta^b$ since $\lambda \geq 1$. This is a contradiction because the equilibrium contract for bad types must satisfy Equation (7). It follows that $\iota^b < m$. □

Recall that Proposition 1 showed that when the price of private insurance is high due to variable administrative costs incurred by the insurer, the optimal contracts will feature less than full insurance for both risk types. Similarly, Proposition 3 shows that when the implicit price of private insurance is high because individuals are at least partially covered by Medicaid than the optimal contracts will also feature less than full insurance.

In region 4 in the graphs in Figure 1, the consumption floor is so high that the good types, who's willingness to pay for private LTCI is lower than the bad types, choose to drop out of the private LTCI market. Notice that, even though the average loads are declining as the consumption floor increases, the load on bad types jumps up upon entry into this region. In regions 1–3, the contracts exhibit cross-subsidization with bad types benefiting from negative loads and good types facing positive loads. In region 4, the insurer is able to make a small amount of positive profits by offering a positive contract that is only attractive to the bad types. Finally, in region 5, Medicaid has a similar effect to that depicted in Figure 2c in the paper. The consumption floor is so large that there are no terms of trade that generate positive profits from either type. The insurer rejects applicants when the consumption floor is in this region as the optimal menus consist of a single $(0, 0)$ contract.

Due to the non-convexities Medicaid creates, conditions (13) and (14) in Proposition 2 are no longer sufficient conditions for rejections to occur, and, although still necessary, are not very useful. Proposition 4 provides a stronger set of necessary conditions for rejections in the presence of Medicaid and a stochastic endowment.

Proposition 4. *If the optimal menu is a $(0, 0)$ pooling contract then*

$$U(\theta^b, \lambda\theta^b\iota, \iota) < U(\theta^b, 0, 0), \quad \forall\iota \in \mathbb{R}_+, \tag{33}$$

and

$$U(\theta^g, \lambda\eta\iota, \iota) < U(\theta^g, 0, 0), \quad \forall\iota \in \mathbb{R}_+. \tag{34}$$

Proof. The proposition is proved by showing that if the conditions don't hold, one can find a menu with at least one nonzero contract that satisfies all the constraints and delivers non-negative profits.

First, assume that condition (33) does not hold but that condition (34) does. If (33) does not obtain, there exists $\iota \in \mathbb{R}_+$ such that

$$U(\theta^b, \lambda\theta^b\iota, \iota) \geq U(\theta^b, 0, 0). \tag{35}$$

Give bad types $(\lambda\theta^b\iota, \iota)$ and good types $(0, 0)$. Under this menu the insurer's profits are

$$\Pi = (1-\psi)\lambda\theta^b\iota + \psi 0 - (1-\psi)\lambda\theta^b\iota - \psi 0 = 0;$$

the participation constraint for the bad types, which is also their incentive compatibility constraint, holds by condition (35); the participation constraint of the good types is trivially satisfied; and the incentive compatibility constraint for the good types is satisfied since

$$U(\theta^g, \lambda\theta^b\iota, \iota) < U(\theta^g, \lambda\eta\iota, \iota) < U(\theta^g, 0, 0),$$

where the first inequality follows from the fact that $\eta < \theta^b$ and the second from condition (34).

Second, assume that condition (34) does not hold which means there exists $\iota \in \mathbb{R}^+$ such that

$$U(\theta^g, \lambda\eta\iota, \iota) \geq U(\theta^g, 0, 0). \tag{36}$$

Give both types $(\lambda\eta\iota, \iota)$. Under this pooling contract the insurer's profits are

$$\Pi = \lambda\eta\iota - \lambda\eta\iota = 0,$$

the participation constraint of the good types holds by condition (36), and the participation constraint for the bad types holds since condition (36) holds and U satisfies the single-crossing property established in Lemma (1). Note that the incentive compatibility constraints are trivially satisfied since both types get the same contract. □

If condition (33) fails, then one can find a profitable contract that bad types would take, and, if condition (34) fails, then one can find a profitable pooling contract that good types would take. The conditions are not sufficient because, while they rule out profitable pooling contracts and separating contracts where good types get no insurance, they do not rule out separating contracts where both types get positive insurance. Absent Medicaid, there can never exist a separating contract that increases profits if the optimal pooling contract is $(0, 0)$. However, the non-convexities introduced by Medicaid break this property. As a result, even when the optimal pooling contract generates negative profits, a profitable separating contract might still exist.

Figure 1 highlights some important distinctions between our model, where contracts are optimal choices of an issuer, and previous research by, for instance, Brown and Finkelstein (2008), Mommaerts (2015), and Ko (2016), who model demand-side distortions in the LTCI market but set contracts exogenously. In regions 2 and 3, notice that Medicaid's presence only impacts the pricing and coverage of the optimal private contracts. In these regions, the insurer responds to the reduced demand for private LTCI by adjusting the terms of the contracts but still offers positive insurance. In contrast, in regions 4 and 5, Medicaid's presence also impacts the fraction of individuals who have any private LTCI. Notice that the Medicaid recipiency rates of both types increase as the consumption floor is increased in these regions. This means that, even though good types do not have LTCI in region 4 and no individuals have it in region 5, Medicaid is covering their NH costs only for a subset of the endowment space. For some realizations of ω, they self-insure. Thus, in these regions, Medicaid is crowding-out demand for private LTCI despite providing only incomplete coverage itself. This crowding-out effect is also present in models with exogenous contracts, however, the effects of Medicaid on the terms of positive contracts is not. Thus, allowing the insurer to adjust the contracts in response to the presence of Medicaid is important because, if the terms of the contracts cannot adjust, then the crowding-out effect of Medicaid on the size of the LTCI market will be overstated.

1.3 Varying Rejection Rates across Risk Groups

The analysis in Sections 3.1 and 3.2 of the paper focuses on the problem of an insurer that offers insurance to a single risk group. We now turn to describe how the extent of rejections changes as we vary observable characteristics of individuals. This discussion provides intuition for the results found using the quantitative model which features an environment with a rich structure of public information and thus multiple risk groups.

One data fact we want the model to account for is that those with lower wealth have lower LTCI take-up rates. An explanation for this observation is that risk groups with low expected endowments are more likely to be rejected by the insurer due to Medicaid. The following proposition formalizes this claim.

Proposition 5. *When $\overline{\omega} - m \leq \underline{c}_{NH}$, the possibility of rejection in equilibrium increases if the distribution of endowments on $[\underline{\omega}, \overline{\omega}]$ is given by $H_1(\cdot)$ instead of $H(\cdot)$ where $H_1(\cdot)$ is first-order stochastically dominated by $H(\cdot)$.*

Proof. It is useful to express Equations (33)–(34) as

$$U(\theta^i, \pi^i, \iota) - U(\theta^i, 0, 0) < 0, \quad \forall \iota \in \mathbb{R}^+, i \in \{g, b\}, \tag{37}$$

where

$$U(\theta^i, \pi^i, \iota) = \int_{\underline{\omega}}^{\overline{\omega}} \Big[\theta^i u(\max(\underline{c}_{NH}, \omega - \pi^i - m + \iota)) + (1-\theta^i) u(\omega - \pi^i)\Big] dH(\omega),$$

with

$$\pi^i = \begin{cases} \lambda \theta^b \iota, & \text{if } i = b, \\ \lambda \eta \iota, & \text{if } i = g. \end{cases}$$

Without loss of generality, assume that $m \geq \iota \geq \pi > 0$.

Let $\Delta U(H)$ and $\Delta U(H_1)$ represent $U(\theta^i, \pi^i, \iota) - U(\theta^i, 0, 0)$ when the endowment distribution is given by $H(\cdot)$ and $H_1(\cdot)$, respectively. Then

$$\Delta U(H) = \int_{\underline{\omega}}^{\overline{\omega}} \tilde{u}(\omega) dH(\omega),$$

and

$$\Delta U(H_1) = \int_{\underline{\omega}}^{\overline{\omega}} \tilde{u}(\omega) dH_1(\omega),$$

where

$$\tilde{u}(\omega) = \Big[\theta^i u(\max(\underline{c}_{NH}, \omega - \pi^i - m + \iota)) + (1-\theta^i) u(\omega - \pi^i)\Big] - \Big[\theta^i u(\max(\underline{c}_{NH}, \omega - m)) + (1-\theta^i) u(\omega)\Big].$$

If $\tilde{u}(\omega)$ is non-decreasing then $\Delta U(H) \geq \Delta U(H_1)$ and rejections are weakly more likely under H_1 than H. When $\overline{\omega} - m \leq \underline{c}_{NH}$ we have

$$\tilde{u}(\omega) = \Big[\theta^i u(\max(\underline{c}_{NH}, \omega - \pi^i - m + \iota)) + (1-\theta^i) u(\omega - \pi^i)\Big] - \Big[\theta^i u(\underline{c}_{NH}) + (1-\theta^i) u(\omega)\Big],$$

and

$$\frac{d\tilde{u}(\omega)}{d\omega} = \begin{cases} \theta^i u'(\omega - \pi^i - m + \iota) + (1-\theta^i)[u'(\omega - \pi) - u'(\omega)], & \omega - \pi^i + \iota > \underline{c}_{NH}, \\ (1-\theta^i)[u'(\omega - \pi^i) - u'(\omega)], & \omega - \pi^i + \iota \leq \underline{c}_{NH}. \end{cases}$$

It is easy to see that $\frac{d\tilde{u}(\omega)}{d\omega} > 0$.

□

It immediately follows from Proposition 5 that the possibility of rejections increases if the expected endowment decreases when $\overline{\omega} - m \leq \underline{c}_{NH}$. When $\overline{\omega} - m > \underline{c}_{NH}$, decreasing the expected endowment may also lead to an increased possibility of rejection. However, in this case, it is also possible that the likelihood of rejections goes down since, absent Medicaid, lowering an individual's endowment raises his demand for insurance.

We also want the model to account for the fact that LTCI take-up rates are declining in frailty and that insurers are more likely to reject frail individuals. In the quantitative model, individuals vary by endowments and frailty, both of which are observable by the insurer, and the distribution of private information varies across these observable types. The following proposition shows two ways of varying the distribution of private information with frailty to generate an increasing possibility of rejection. Note that the proof is for the no Medicaid case, although, as the quantitative results illustrate, the proposition holds even when Medicaid is present.

Proposition 6. *When $\lambda > 1$ and θ^b is sufficiently close to 1, the possibility of rejection in equilibrium increases if:*

1. *θ^b increases;*
2. *θ^b increases and θ^g decreases such that the mean NH entry probability $\eta \equiv \psi\theta^g + (1-\psi)\theta^b$ does not change.*

Proof. Without Medicaid, rejection will occur in equilibrium iff

$$f^b(\theta^b) \equiv \lambda\theta^b - MRS(\theta^b, 0, 0) \geq 0, \tag{38}$$

and

$$f^g(\theta^g, \theta^b) \equiv \lambda\eta - MRS(\theta^g, 0, 0) \geq 0, \tag{39}$$

where $\eta \equiv \psi\theta^g + (1-\psi)\theta^b$.

1. Differentiating f^b with respect to θ^b yields

$$\frac{f^b(\theta^b)}{d\theta^b} = \lambda - \frac{\int_{\underline{\omega}}^{\overline{\omega}} u'(\omega - m)dH(\omega) \int_{\underline{\omega}}^{\overline{\omega}} u(\omega)dH(\omega)}{[\theta^b \int_{\underline{\omega}}^{\overline{\omega}} u'(\omega - m)dH(\omega) + (1-\theta^b)\int_{\underline{\omega}}^{\overline{\omega}} u'(\omega)dH(\omega)]^2}. \tag{40}$$

When $\theta^b = 1$, Equation (40) is positive since

$$\frac{f^b(\theta^b)}{d\theta^b}|_{\theta^b=1} = \lambda - \frac{\int_{\underline{\omega}}^{\overline{\omega}} u'(\omega)dH(\omega)}{\int_{\underline{\omega}}^{\overline{\omega}} u'(\omega - m)dH(\omega)},$$

$\lambda \geq 1$ and $u'(\omega) < u'(\omega - m)$ for $m > 0$. It is easy to see that Equation (40) is increasing in θ^b. Thus if θ^b is sufficiently close to 1, increasing θ^b will increase f^b. Differentiating f^g with respect to θ^b yields

$$\frac{f^g(\theta^g, \theta^b)}{d\theta^b} = \lambda(1 - \psi) > 0.$$

Thus increasing θ^b increases f^g.

2. The proof that f^b is increases is the same as in 1 since f^b does not depend on θ^g. The first term of f^g does not change. The second term only depends θ^g and differentiating it with respect to θ^g yields

$$\frac{dMRS(\theta^g, 0, 0)}{d\theta^g} = \frac{\int_{\underline{\omega}}^{\overline{\omega}} u'(\omega - m) dH(\omega) \int_{\underline{\omega}}^{\overline{\omega}} u(\omega) dH(\omega)}{[\theta^g \int_{\underline{\omega}}^{\overline{\omega}} u'(\omega - m) dH(\omega) + (1 - \theta^g) \int_{\underline{\omega}}^{\overline{\omega}} u'(\omega) dH(\omega)]^2} > 0.$$

So f^g increases when θ^g declines.

□

Proposition 6 presents two ways to increase the possibility of a no-trade equilibrium occurring. Either of the two ways can, in theory, be used to generate decreasing LTCI take-up rates with frailty. If, as in way 1, only θ^b increases then both the mean and the dispersion of the NH entry probabilities will increase. However, way 2 states that increasing the dispersion of entry probabilities while holding the mean fixed by varying both θ^b and θ^g can also generate increased rejection rates. In short, to generate an increase in rejection rates with frailty, both ways require an increase in the dispersion in NH entry probabilities with frailty. However, way 1 also requires an increase in the mean.

The fact that both ways of increasing the rejection rates requires an increase in dispersion of private information is consistent with the empirical findings of Hendren (2013) that adverse selection is more severe among individuals that are more likely to rejected by LTC insurers. In addition, we show in Section 4 in the paper that the increase in dispersion is consistent with the pattern of NH entry probabilities in the data, but the increase in the mean implied by case 1 is inconsistent. Thus both θ^b and θ^g must vary with frailty, as in case 2, to generate patterns of both LTCI take-up rates and NH entry probabilities that are consistent with the data. Note that Hendren (2013) also presents a theory of how private information can generate no-trade equilibria (rejections). His mechanism, however, is different from ours. We generate no-trade equilibria by modeling administrative costs on the insurer and Medicaid. In his model, there is a continuum of private types and he allows there to be a positive mass of individuals who have probability 1 of incurring the loss. He shows that, under this assumption, the presence of private information can lead to no-trade equilibria. To activate his mechanism in our model with 2 private types we would have to assume that $\theta^b = 1$. He also shows that the possibility of rejections increases as the magnitude of private information increases. In our model, an increase in the magnitude of private information would be equivalent to an increase in $\theta^b - \theta^g$ and, with $\theta^b < 1$, does not necessarily increase the possibility of rejection. For example, increasing $\theta^b - \theta^g$ and at the same time lowering both θ^b and θ^g can reduce the probability of rejection.

1.4 Variable costs proportional to claims versus premia

When variable administrative costs that are proportional to claims are present the insurer's maximization problem is given by Equations (1)–(3) with $k = 0$ and the first-order conditions are given by Equations (6)–(7). Now suppose instead that the variable administrative costs are proportional to premia. The insurer's maximization problem becomes

$$\max_{\pi^i,\iota^i} \psi[\delta\pi^g - \theta^g\iota^g] + (1-\psi)[\delta\pi^b - \theta^b\iota^b] \tag{41}$$

subject to

$$(PC_i) \quad U(\theta^i,\pi^i,\iota^i) - U(\theta^i,0,0) \geq 0, \quad i \in \{g,b\}, \tag{42}$$
$$(IC_i) \quad U(\theta^i,\pi^i,\iota^i) - U(\theta^i,\pi^j,\iota^j) \geq 0, \quad i,j \in \{g,b\},\ i \neq j, \tag{43}$$

where $0 \leq \delta < 1$ reflects the costs. In this case, when Equation (42) binds for the good types and Equation (43) binds for the bad types the optimal menu must satisfy

$$\psi MRS(\theta^g,\pi^g,\iota^g) + (1-\psi)\left[\frac{U_\pi(\theta^b,\pi^g,\iota^g)}{U_\pi(\theta^b,\pi^b,\iota^b)}MRS(\theta^g,\pi^g,\iota^g) + \frac{U_\iota(\theta^b,\pi^g,\iota^g)}{U_\pi(\theta^b,\pi^b,\iota^b)}\right] = \psi\theta^g/\delta, \tag{44}$$
$$MRS(\theta^b,\pi^b,\iota^b) = \theta^b/\delta. \tag{45}$$

Notice that when $\lambda = 1/\delta$ the systems of equations that determine the optimal menus in these two scenarios are equivalent.

1.5 Adding more periods to the quantitative model

In this section we show that period 1 of our 3 period model can easily be replaced with multiple periods in which the young make consumption and savings decisions at annual frequencies. To simplify the analysis we abstract from initial differences in frailty and assume that the entire working-age endowment is received when individuals retire. In our three period model, working-aged individuals face no risks. Thus, the essence of the savings decision of a working-aged person in our model is captured by the following two period consumption-savings problem for an individual where for convenience we assume that the endowment, ω_o is received at the start of the second and final period of life, $V(a_o)$ is the value function of an individual at the point of retirement, and E is the expectations operator.

$$\begin{aligned}&\max_{c_y,a_o} \frac{c_y^{1-\sigma}}{1-\sigma} + \beta EV(a_o)\\ &s.t.\\ &c_y + a_o/R = \omega_o.\end{aligned}$$

The FONC is:

$$c_y^* = R\beta EV'(a_o).$$

and combining the FONC with the budget constraint yields the following expression for a_o:

$$a_0 = \left(\omega_o - [R\beta EV'(a_o)]^{-1/\sigma}\right) R.$$

Suppose instead that an individual works for J years before retiring. The problem is given by

$$\max_{\{c_j\}_{j=1}^{J},\hat{c}_0} \sum_{j=1}^{J} \gamma^{j-1} \frac{c_j^{1-\sigma}}{1-\sigma} + \gamma^J EV(a_o)$$

$$s.t.$$

$$\sum_{j=1}^{J} c_j/\hat{R}^{j-1} + \hat{a}_o/\hat{R}^J = \omega_o.$$

The FONCs for this problem are:

$$\hat{R}\gamma EV'(a_o) = c_J^{-\sigma},$$
$$c_1^{-\sigma} = c_j^{-\sigma}(\gamma\hat{R})^{j-1}.$$

Using these expressions we can express a_o as:

$$a_0 = \left[\omega_o - \sum_{j=1}^{J} \frac{[\gamma\hat{R}]^{\frac{j-1}{\sigma}}}{\hat{R}^{j-1}} \left((\hat{R}\gamma)^J EV'(a_o)\right)\right] \hat{R}^J. \quad (46)$$

Next note that if there are 40 years of youth then $\gamma = \beta^{1/J}$ and, for a given R, Equation (46) can be solved to find the corresponding value of $\hat{R}$ that delivers the same assets at retirement, a_o, in the two problems.

2 Details of the data work

Our HRS sample is constructed from the 1992 to 2012 waves of HRS and AHEAD. The sample is essentially the same as Braun et al. (2015) and Kopecky and Koreshkova (2014). Beyond adding additional data from 1992, 1994, and 2012, there are a few other changes. There is no censoring at -500 and 500 for asset values near 0. We assign an individual's 1998 weight (or post-1998 mean weight, if their 1998 weight is 0) to pre-1998 waves where their weight is 0. The main definitional novelties/changes are now provided. An individual is retired if his labor earnings are less than \$1500 (in 2000 dollars). An individual is considered to have ever had long-term care insurance if they report having been covered in half or more of their observed waves.

Nursing home event A nursing home event occurs when an individual spends 100 days or more in a nursing home within the approximately two year span between HRS interviews or within the period between their last interview and death. If the individual dies less than 100 days after their last interview, but at the time of their death had been in a nursing home for over 100 days, this also counts as a nursing home event. In the HRS, there are several (sometimes inconsistent) variables that provide information about the number of days spent in a nursing home. From the RAND dataset, we use the total nursing nights over all stays during the wave, as well as the number of days one has been in a nursing home (conditional on being in a nursing home at the time of the interview). This information is also pulled from the exit data, as well as the date of entry to a nursing home, provided the individual died there. Interview and death dates are used when a respondent reports having been continuously in a nursing home since the previous wave. Since the information is sometimes conflicting, and one piece often missing when another observed, a nursing home event is assigned if any of the variables suggest a person met the criteria described above.

Permanent Income To calculate permanent income, first sum the household heads social security and pension income and average this over all waves in which the household head is retired. The cumulative distribution of this average is defined as the permanent income, which ranges from 0 to 1. For singles, the household head is the respondent, and for couples it is the male.

Wealth We use the wealth variable ATOTA which is the sum of the value of owned real estate (including primary residence), vehicles, businesses, IRS/Keogh accounts, stocks, bonds, checking/savings accounts, CDs, treasury bills and "other savings and assets" less any debt reported.

2.1 Rejections

Many applications for LTCI are rejected. Murtaugh et al. (1995) in one of the earliest analyses of LTCI underwriting estimates that 12–23% of 65 year olds, if they applied, would be rejected by insurers because of poor health. Their estimates are based on the National Mortality Followback Survey. Since their analysis, underwriting standards in the LTCI market have become more strict. We estimate rejection rates of between 36% and 56% for 55–65 year olds by applying underwriting guidelines from Genworth and Mutual of Omaha to a sample of HRS individuals.

To understand how we arrive at these figures, it is helpful to explain how LTCI underwriting works. Underwriting occurs in two stages. In the first stage, individuals are queried about their prior LTC events, pre-existing health conditions, current physical and mental capabilities, and lifestyle. Some common questions include: Do you require human assistance to perform any of your activities of daily living? Are you currently receiving home health care or have you recently been in a NH? Have you ever been diagnosed with or consulted a medical professional for the following: a long list of diseases that includes diabetes, memory loss, cancer, mental illness, heart disease? Do you currently use or need any of the following: wheelchair, walker, cane, oxygen? Do you currently receive disability benefits, social security disability benefits, or Medicaid?[1] A positive answer to any one of these questions is sufficient for the insurers to reject applicants before they have even submitted a formal application. Many of these same questions are asked to HRS participants. As Table 1 shows, the fraction

[1]Source: 2010 Report on the Actuarial Marketing and Legal Analyses of the Class Program

Table 1: Percentage of HRS respondents who would answer "Yes" to at least one LTCI prescreening question.

	Age		
	55–56	60–61	65–66
All	41.8	43.7	49.5
Top Half of Wealth Distribution Only	30.8	33.6	39.3

Data source: Authors' calculations using our HRS sample.

of individuals in our HRS sample who would respond affirmatively to at least one question is high and ranges from 40.5–to 49.6% depending on age. Rejection rates are also high in the top half of the wealth distribution ranging from 30.8% –to 39.3%. Question 3 pertaining to previously diagnosed diseases received the highest frequency of positive responses. If we are conservative and omit question 3 the prescreening declination rate ranges from 17.5–22.5% for all individuals and from 10.0% –12.1% for individuals in the top half of the wealth distribution.

If applicants pass the first stage, they are invited to make a formal application. Medical records and blood and urine samples are collected and the applicants cognitive skills are tested. One in five formal applications are denied coverage based on industry surveys (see Thau et al., 2014). Assuming a 20% rejection rate for the second round, the overall rejection rate is 55.6% for 55–66 years old in our HRS sample using all questions and 35.9% if question three is omitted. For individuals in the top half of the wealth distribution the rejection rates are 47.7% and 28.0% respectively.

2.2 Construction of the Frailty Index

Table 2 lists the variables we used to construct the frailty index for HRS respondents. The choice of these variables is based on Genworth and Mutual of Omaha LTCI underwriting guidelines. To construct the frailty index, first sum the variables listed in the first column of Table 2, assigning each a value according to the second column. Then divide this sum by the total number of variables observed for the individual in the year, as long as the total includes 30 or more variables. The construction of this frailty index mostly follows the guidelines laid out in Searle et al. (2008), and uses a set of HRS variables similar to the index created in Yang and Lee (2009). There are a couple of differences however. Primarily, a few variables that do not necessarily increase with age (e.g. drinking $>$ 15 drinks per week and smoking) were included. Also, cognitive tests are broken into parts which each count as separate variables, essentially increasing their weight in the index relative to Searle et al. (2008), which uses only a single variable for cognitive impairment. Nevertheless, our frailty distribution still closely resembles those of frailty indices used in other papers.

Table 2: Health Variables for Frailty Index Construction

Variable	Value
Some difficulty with ADL/IADLs:	
Eating	Yes=1, No=0
Dressing	Yes=1, No=0
Getting in/out of bed	Yes=1, No=0
Using the toilet	Yes=1, No=0
Bathing/shower	Yes=1, No=0
Walking across room	Yes=1, No=0
Walking several blocks	Yes=1, No=0
Using the telephone	Yes=1, No=0
Managing money	Yes=1, No=0
Shopping for groceries	Yes=1, No=0
Preparing meals	Yes=1, No=0
Getting up from chair	Yes=1, No=0
Stooping/kneeling/crouching	Yes=1, No=0
Lift/carry 10 lbs	Yes=1, No=0
Using a map	Yes=1, No=0
Taking medications	Yes=1, No=0
Climbing 1 flight of stairs	Yes=1, No=0
Picking up a dime	Yes=1, No=0
Reaching/ extending arms up	Yes=1, No=0
Pushing/pulling large objects	Yes=1, No=0
Cognitive Impairment:	
Immediate Word Recall	+.1 for each word not recalled (10 total)*
Delayed Word Recall	+.1 for each word not recalled (10 total)*
Serial 7 Test	+.2 for each incorrect substraction (5 total)
Backwards Counting	Failed test=1, 2nd attempt = .5, 1st attempt = 0
Identifying obejcts & Pres/VP	.25 for each incorrect answer (4 total)
Identifying date	.25 for each incorrect answer (4 total)
Ever had one of following conditions:	
High Blood Pressure	Yes=1, No=0
Diabetes	Yes=1, No=0
Cancer	Yes=1, No=0
Lung disease	Yes=1, No=0
Heart disease	Yes=1, No=0
Stroke	Yes=1, No=0
Psychological problems	Yes=1, No=0
Arthritis	Yes=1, No=0
BMI ≥ 30	Yes=1, No=0
Drinks 15+ alcoholic drinks per week	Yes=1, No=0
Smokes Now	Yes=1, No=0
Has smoked ever	Yes=1, No=0

*For the 1994 HRS cohort, 40 questions were asked (instead of 20) for word recall. In this year, each missed question receives weight 0.05.

Table 3: LTCI take-up rates by wealth and frailty for married and not married individuals

	Married					Not Married				
Frailty	Wealth Quintile					Wealth Quintile				
Quintile	1	2	3	4	5	1	2	3	4	5
1	0.011	0.044	0.095	0.151	0.242	0.010	0.093	0.108	0.134	0.205
2	0.031	0.057	0.119	0.156	0.219	0.012	0.017	0.047	0.163	0.175
3	0.011	0.027	0.084	0.141	0.212	0.031	0.038	0.091	0.092	0.174
4	0.014	0.036	0.068	0.098	0.178	0.016	0.021	0.069	0.145	0.127
5	0.026	0.026	0.040	0.095	0.076	0.007	0.032	0.066	0.122	0.145

For frailty (rows) Quintile 5 has the highest frailty and for wealth (columns) Quintile 5 has the highest wealth. Data source: 62–72 year olds in our HRS sample.

2.3 Evidence of private information

Hendren (2013) finds that self-assessed NH entry risk is only predictive of a NH event for individuals who would likely be rejected by insurers. Hendren's measure of a NH event is independent of the length of stay. Since we focus on stays that are at least 100 days, we repeat the logit analysis of Hendren (2013) using our definition of a NH stay and our HRS sample. We get qualitatively similar results. We restrict the sample to individuals ages 65–80. We find evidence of private information at the 10 year horizon (but not at the 6 year) in a subsample of this sample consisting of individuals who would likely be rejected by insurers. This sample includes individuals who have any ADL/IADL restriction, past stroke, or past nursing or home care. The p-value for a Wald test which restricts the coefficients on subjective probabilities to zero is 0.003 at the 10 year horizon and 0.169 at the 6 year. If all individuals above age 80 are included in the reject sample as well the p-values at both horizons are less than 0.000. For a sample of individuals who would likely not be rejected we are unable to find evidence of private information. The p-value for a Wald test which restricts the coefficients on subjective probabilities to zero is 0.210 at the 10 year horizon and 0.172 at the 6 year.

2.4 LTCI take-up rate patterns controlling for family status

Tables 3 and 4 shows LTCI take-up rates of by frailty and wealth quintiles for married versus single individuals and individuals with and without children. The general pattern of take-up rates by frailty and wealth are robust to controlling for martial status and children. LTCI take-up rates increase with wealth and decline with frailty for both married and single individuals and for both individuals with and those without children. Comparing the levels of take-up rates across married and single individuals shows that in many wealth and frailty quintiles there is no systematic difference between them. The only discernible differences between individuals with and without children are that wealthy individuals without children, those in quintiles 4 and 5 of the wealth distribution, tend to have slightly higher take-up rates then those with children.

Table 4: LTCI take-up rates by wealth and frailty for individuals with and without children

	Have Children					Do Not Have Children				
Frailty	Wealth Quintile					Wealth Quintile				
Quintile	1	2	3	4	5	1	2	3	4	5
1	0.012	0.058	0.100	0.146	0.221	0.000	0.059	0.076	0.186	0.337
2	0.017	0.051	0.103	0.153	0.196	0.061	0.000	0.064	0.238	0.314
3	0.022	0.033	0.085	0.132	0.202	0.022	0.000	0.088	0.071	0.214
4	0.017	0.033	0.067	0.106	0.142	0.000	0.021	0.130	0.178	0.271
5	0.017	0.025	0.043	0.101	0.101	0.001	0.021	0.239	0.132	0.173

For frailty (rows) Quintile 5 has the highest frailty and for wealth (columns) Quintile 5 has the highest wealth. Data source: 62–72 year olds in our HRS sample.

2.5 Description of the auxiliary simulation model

To obtain survival and lifetime NH entry probabilities by frailty and PE quintile groups, we use an auxiliary simulation model similar to that in Hurd et al. (2013). First, using a multinomial logit, we estimate transition probabilities between four states that we observe in the HRS: alive and dead, each with and without a nursing home event in the last two years. These probabilities depend on age, PE, NH event status, and frailty, including polynomials and interactions of these variables. Specifically, age is modeled as a cubic function, frailty as a quadratic, and the others are both linear. Interactions include age with each of the other first-order terms, as well as frailty with PE. We also simulate lifetime frailty paths because we need them since, in contrast to Hurd et al. (2013), we include frailty in the multinomial logit. This is done using estimates from a fixed effects regression of frailty on lagged frailty, age, and age squared.

Simulations begin at age 67. To get the initial distribution of explanatory variables, we first average frailty and population weights across all observations at which an individual is between ages 62-72. PE and the estimated fixed effect are constant within individuals. The initial distribution then draws 500,000 times from this person-level weighted distribution. The model simulates two-year transitions, following the structure of the HRS data, and assigns age of death by randomly choosing an age between their last living wave and the death wave.[2]

3 Computation

Computing an equilibrium in our model is subtle because Medicaid NH benefits are means-tested and Medicaid is a secondary payer of NH benefits. Individual saving policies exhibit

[2]Note that a half year is added to death age to account for the fact that reported ages are the floor of a respondent's continuous age. Nursing home entry ages are similarly assigned, but we add only 0.2 years due to the 100 day requirement of a nursing home event. They are also upwardly bound by death age when both occur in the same wave.

jumps and the demand for private insurance interacts in subtle ways with $q(\kappa)$, the distribution of consumption demand shocks.

We start by discretizing the endowment and frailty distributions. The number of grid points for endowments $\mathbf{w}$ is $ny = 101$ and frailty takes on $nf = 5$ grid points. The consumption demand shock κ is also discretized: $nk = 50$.

The specific algorithm for computing an equilibrium proceeds as follows. First, we guess values for profits (which gives us dividends) and taxes and then we iterate over profits and taxes until profits converge and taxes satisfy the government budget constraint. In each iteration, we have to solve for allocations, contracts, and profits for each combination of endowments and frailty in the discretized state space. For each point in the discertized state space $(\mathbf{w}_s, f_j)$, $s = 1, nw$ and $j = 1, nf$, we guess a level of savings: $\hat{a}_{f_j,\mathbf{w}_s}$. Given $\hat{a}_{f_j,\mathbf{w}_s}$, we then solve for the optimal contracts as follows. The optimal contract for a risk group depends on which individuals of observable type $(\mathbf{w}_s, f_j)$ qualify for Medicaid if they incur the NH shock. Thus, it depends on the specific combinations of the κ shock and the private type $i \in \{g, b\}$ that imply that an individual qualifies for Medicaid. Because of the non-convexities introduced by Medicaid the Kuhn-Tucker conditions of the insurer's problem are not sufficient. However, if one first assumes a distribution of individuals across Medicaid, then a contract satisfying the Kuhn-Tucker conditions is sufficient. So we solve for the optimal contract for all feasible combinations of individuals with different κ's and i's receiving Medicaid. The number of cases that has to be considered is large but it can reduced by the noting that for a given value of κ if a bad type is on Medicaid the good type is also on Medicaid by the single-crossing property and that if a type i qualifies for Medicaid for a value of κ he will also qualify for Medicaid for all larger values of κ.

To solve for the optimal contracts for each Medicaid distribution, first we solve for the optimal pooling contract. Second, we check to see if an optimal separating menu exists. The contract of type g under the optimal separating menu is the same as the optimal pooling contract. So we fix the good type's contract at the optimal pooling one and solve for the optimal separating contract of the bad type (if it exists).[3] The optimal contract for observable type $(\mathbf{w}_s, f_j)$ under the current guess for savings, $\hat{a}_{f_j,\mathbf{w}_s}$ is then the one that maximizes the insurer's profits. Finally, we iterate over savings until we find the value of savings that maximizes expected lifetime utility.

4 Additional Calibration Details

Table 5 list the values of many of the model parameters. The survival probabilities of each frailty and PE quintile in the quantitative model are shown in the left panel of Figure 2. The right panel shows the mean lifetime NH entry probability conditional on surviving for each frailty/PE quintile combination in the model. These NH entry probabilities in the model match those in the data because we parametrized the model to reproduce the survival probabilities in the left panel and the unconditional NH entry probabilities in Figure 4 in the paper.

[3] If a separating menu doesn't exist it means one of the inequality constraints is violated.

Table 5: Model Parameters

Description	Parameter	Value
Risk aversion coefficient	σ	2
Preference discount factor	β	0.94
Retirement preference discount factor	α	0.20
Interest rate (annualized)	r	0.00
Frailty distribution	h	BETA(1.54,6.30)
Young endowment distribution	w_y	$\ln(w_y) \sim \mathcal{N}(-0.32, 0.64)$
Copula parameter	ρ_{f,w_y}	-0.29
Demand shock distribution	κ	$1-\kappa \sim$ truncated log-normal
Demand shock mean	μ_κ	0.6
Demand shock standard deviation	σ_κ	0.071
Fraction of good types	ψ	0.709
Nursing home cost	m	0.0931
Insurer's variable cost of paying claims	λ	1.195
Insurer's fixed cost of paying claims	k	0.019
Medicaid consumption floor	$\underline{c}_{NH}$	0.01855
Welfare consumption floor	$\underline{c}_o$	0.01855

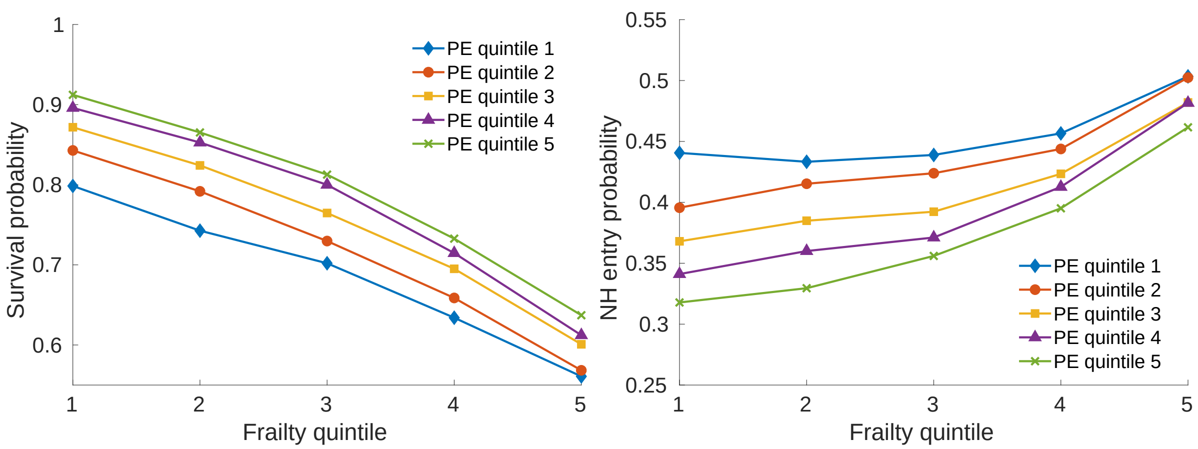


Figure 2: The probability of surviving to age 80 or until experiencing a NH stay (left panel) and the probability that a 65-year old will enter a NH conditional on surviving to age 80 (right panel) by frailty and PE quintile. The probabilities are based on our auxiliary simulation model which is estimated using HRS data. NH entry probabilities are for a NH stay of at least 100 days.

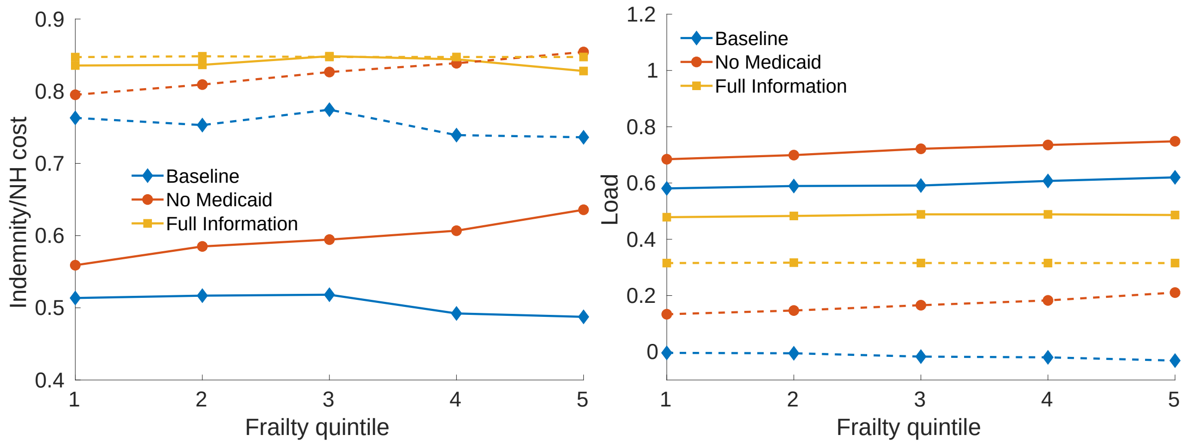


Figure 3: Insurance coverage and loads by frailty quintile.
The left panel reports LTCI indemnities relative to medical costs of a NH stay and the right panel reports loads for the two private information types: good risks (solid lines) and bad risks (dashed lines). Three economies are reported: Baseline, Full Information and No Medicaid.

Determination of nursing home cost m. We estimate the average medical and nursing expense component of NH costs as follows. First, we use data from the Minnesota Office of the Legislative Auditor (1995) which provides a breakdown of SNF and RCC costs for 5 midwest states in 1994. We adjust each cost in the breakdown using either the CPI or the medical CPI to create a cost breakdown for the year 2000. Then we calculate the share of costs due to medical and nursing expenses and the share due to room and board for each state and average them across states using state population weights. The population weights are taken from the 2000 U.S. Census. We find that, on average, 76% of SNF and RCC costs are due to medical and nursing expenses and 24% are room and board. Next, we obtain estimates of the average annual total costs of SNF and RCC stays in the U.S. in 2000 of $60,000 and $28,099, respectively, from Stewart et al. (2009). Using these and the shares, we calculate the average annual cost of the medical and nursing expense component of each type of stay. Finally, we average the annual cost of the medical and nursing expense component across NH and RCC stays using data from Spillman and Black (2015) on the fraction of individuals in residential care who are in RCC's versus SNF's. We obtain an average medical and nursing expense component of residential LTC costs of $32,844 per annum in year 2000. Braun et al. (2015) estimate that the average duration of NH stays that exceed 90 days is 3.25 years. Medicare provides NH benefits for up to the first 100 days. To account for this, we subtract 100 days resulting in an average benefit period of 2.976 years. Multiplying the annual cost by the average benefit period yields total medical and nursing costs of a NH stay of $97,743 or a value of $m = 0.0931$ when scaled by average lifetime earnings.

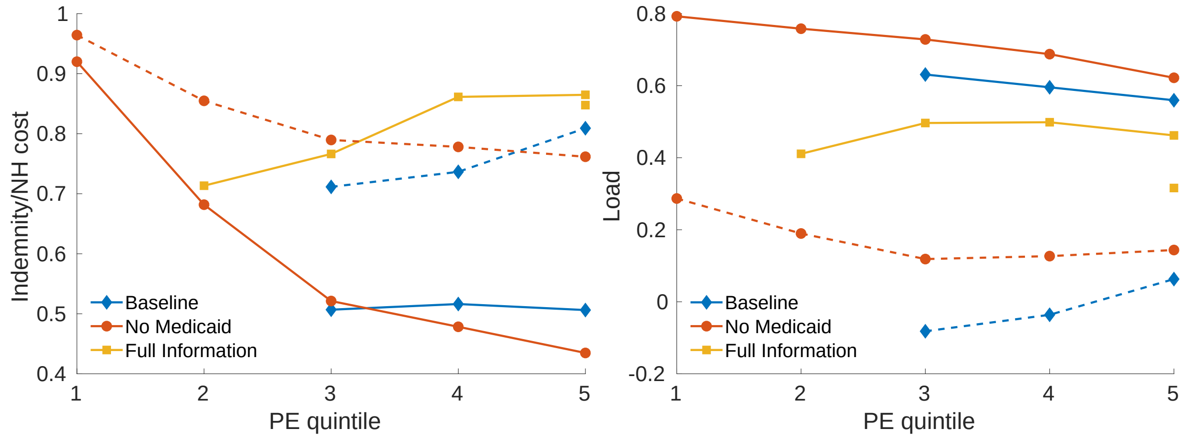


Figure 4: Insurance coverage and loads by PE quintile.
The left panel reports indemnities relative to the medical cost of a NH stay and the right panel reports loads for the two private information types: good risks (solid lines) and bad risks (dashed lines). Three economies are reported: Baseline, Full Information and No Medicaid.

5 Additional Results

5.1 Pricing and Coverage in Baseline and Other Economies

Figures 3 and 4 show how Medicaid and adverse selection influence pricing and coverage at alternative frailty and PE levels. Removing private information increases the coverage of both types relative to coverage in the Baseline and reduces the variation in loads. Reducing the scale of Medicaid also increases the level of coverage for both private information types at each frailty quintile. However, the loads are also higher. Notice also that coverage increases monotonically in frailty in the No Medicaid economy.

Figure 4 reports how coverage and loads vary by PE quintile for the same three scenarios. In the Full Information economy bad risks in PE quintiles 1–4 do not get positive private insurance. Coverage of good risks in increasing in PE and higher than in the Baseline. Loads on good risks are lower than in Baseline and humped-shape in PE. In the No Medicaid economy, as PE increases, bad and good risk types experience lower coverage and slightly declining loads. Reducing the Medicaid NH benefit floor has a very big impact on the poor. Their demand for LTCI is inelastic and, as a result, they now face the highest loads but also receive the most coverage.

5.2 Robustness Analysis

Size of the Medicaid Consumption Floor Our Baseline parameterization of the model uses the same Medicaid consumption floor as Brown and Finkelstein (2008). They note that their results are sensitive to the size of this parameter and other empirical work has sometimes used higher consumption floors. To assess the robustness of our conclusions to the scale of Medicaid we recalibrated the model positing a Medicaid consumption floor that is 1.76 times larger than the value in the Baseline economy. This value is at the high end of previous estimates (see Kopecky and Koreshkova (2014) for a summary of consumption floor

Table 6: Robustness: Rejection rates in the Baseline, the No Administrative Costs, the No Medicaid, and the Full Information Economies with Higher Consumption Floors.

Scenario Description	Baseline	No Admin. Costs $\lambda = 1$, $k = 0$	No Medicaid $\underline{c}_{nh} = 0.001$	Full Information $\theta^i_{f,\mathbf{w}}$ public
Average	90.1	39.0	9.9	48.3
By PE Quintile				
1	100	100	15.4	100
2	100	83.3	0.0	99.8
3	82.5	0.0	0.0	31.8
4	86.8	0.0	0.0	0.0
5	75.8	0.0	0.18	50.0
By receiving Medicaid NH benefits conditional on surviving				
Would	44.1	35.7	3.1	41.1
Would not	44.9	1.0	0.0	15.2

Rejection rates are percentage of individuals who are only offered a single contract of $(0, 0)$ by the insurer. Note that for PE quintiles that the figures are expressed as a percentage of individuals in that quintile. However, the bottom two rows of the table are a decomposition of the average rejection rate for that economy.

values). Table 6 indicates that Medicaid now has a bigger impact on producing rejections among affluent households. Rejection rates are only 18% in wealth quintile 5 if Medicaid is removed. However, administrative costs and private information continue to be very important among more affluent individuals. Removing either friction still has a very large impact on rejection rates in PE quintiles 3-5. Rejection rates fall to zero in these groups if administrative costs are removed and they fall by 50% or more if private information is removed. In addition, supply-side frictions continue be responsible for the large fraction of individuals who pay for NH expenses out-of-pocket in the Baseline.

How well can a model do that abstracts from private information? We have found that the Full Information economy has very high LTCI take-up rates among higher wealth quintiles and produces an incorrect pattern of take-up rates by frailty in wealth quintiles 4 and 5. Is there a way to remedy these issues with the Full Information economy by re-calibrating it? To explore this possibility we recalibrated the Full Information economy so that it reproduces the average LTCI take-up rate by increasing fixed and variable administrative costs in a proportionate fashion. The resulting magnitude of the administrative costs increased from 32.6% of premium to 49% of premium. The Full Information economy with higher administrative costs continues to have problems reproducing the pattern of LTCI take-up rates by wealth and frailty quintile. For instance, LTCI take-up rates in wealth quintile 5 are now zero. From the perspective of this group these administrative costs are so high that they prefer to self-insure NH risk. LTCI take-up rates in wealth quintile 4 are positive. However, they are not declining in frailty as occurs in our HRS data (See

Table 2 in the paper). We also explored lowering the LTCI take-up rates in this economy by varying the θ's. However, we could not generate enough variation in the θ's to reproduce the average level of LTCI take-up we see in the data nor did varying the θ's help us reproduce the empirical pattern of LTCI take-up rates by frailty in the top two wealth quintiles.

How well can a model do that abstracts from administrative costs? We have also investigated whether a version of the model with no administrative costs could hit our calibration targets. It is also a challenge for the model to reproduce low LTCI take-up rates in high wealth quintiles when administrative costs are zero. Consider, for instance, the group in wealth and frailty quintile 5. When administrative costs are set to zero the LTCI take-up rate for this group increases from 0.118 to 0.99. if the nursing home entry rate for bad risks is increased to one ($\theta^b = 1$) the LTCI take-up rate falls to 0.96. This model also predicts a high LTCI take-up rate for this group if assume that one half of all individuals are bad risks $\psi = 0.5$ and have NH entry probabilities of one. The model produces a LTCI take-up rate of 0.44 while the LTCI take-up rate for this group in our dataset is only 0.104.

Taken together these results suggest that both private information and administrative costs are required if the model is to produce LTCI take-up rates that have the same magnitude and pattern across different wealth and frailty quintiles as our data.

Insurers do pay out at the other end. One reason that has been offered for low LTCI take-up rates is that people are concerned that insurers will come up with reasons for not paying out at the time the NH event occurs (see, for instance, Duhigg (2005)). However, survey evidence suggests that most individuals are happy with the claims filing experience. A survey conducted in 2015–2016 by LifePlans Inc., a service provider for insurers, found that 78 percent of claimants found it easy to file a claim.[4] Only 6% had a disagreement with their company about coverage and disagreements in a majority of cases were resolved in favor of the policy holder. Taken together they find that only 2% claims filers find themselves in a situation where they disagree with their insurer and the problem is not resolved to their satisfaction. Another survey commissioned by the U.S. Department of Health and Human Services in 2007 produced similar results (See U.S. Department of Health and Human Services (2007)). It finds that benefits were approved for 95.7% of respondents filing claims and that of those initially denied benefits more than half subsequently received benefits in the ensuing 12 month period.

Multiple sources of private information and heterogeneous preference discount rates. We have found that our model with a single source of private information can account for a broad range of empirical regularities in the U.S. LCTI market including the correlation puzzle. This is not to say that insurers do not have to contend with private preference heterogeneity in risk or preference discount rates in this market. Our analysis suggests that these considerations may not be of first order importance to issuers given the institutional features in the U.S. market that we have modeled. To provide a specific example we multiple sources of private information may not be of central importance here, consider private differences in preference discount rates. Higher preference discount rates among

[4]See Lifeplans, Inc. (2016).

frail could possibly help account for rejections among poorer individuals but, Medicaid is very already very effective in producing rejections among poorer and even middle class individuals. So it is not clear that there is a need to appeal to private differences in discount rates to produce rejections in these groups. It is also not clear that modeling a positive correlation between frailty and privately observed preference discount rates would be help us in accounting for rejections among wealthy frail individuals. The first order implication of a high preference discount rate is to save less and consume more and one would thus expect that frail individuals in high wealth quintiles are reasonably patient.

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