Economics 613/614

Fall 2007

Assignment 1

1. (a) Write a Fortran subroutine that finds the minimum of a one-dimensional function using the Golden Search Method.

   (b) Write a program to test your procedure.

2. (Use Matlab for this problem.) Consider the function

$$f(x) = \frac{1}{1+25x^2}$$

defined on the interval $[0,1]$, the functions

$$g_\sigma(x) = \frac{x^{1-\sigma}-1}{1-\sigma}$$

defined on the interval $[0.01,1]$ and the functions

$$v_{\alpha,\delta,\sigma}(x) = \frac{[x^\alpha + (1-\delta)x]^{1-\sigma}-1}{1-\sigma}$$

defined on $[0.01, \delta^{\frac{1}{\alpha-1}}]$.

   (a) Approximate f by a function

$$\widehat{f}_n(x) = a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n,$$

   choosing the coefficients a_k so that $\widehat{f}(x_i) = f(x_i)$ for each $i = 0, 1, \ldots, n$. The x_i's should constitute an evenly spaced grid, i.e. $x_i = i/n$. Define the relative error function $r(n)$ via

$$r(n) = \frac{\|\widehat{f}_n(x) - f(x)\|}{\|f(x)\|}$$

   where, for any function $h : X \to \mathcal{R}$,

$$\|h(x)\| = \sup_{x\in X} |h(x)|.$$

   Plot the function $h_n(x) = \widehat{f}_n(x) - f(x)$ function and compute $r(n)$ for some values of n. Is h_n equioscillatory? Is there any tendency for $r(n)$ to shrink towards zero as n increases?

(b) Approximate f by a Chebyshev polynomial of degree n with $m = n + 1$ Chebyshev nodes. Repeat the exercise in (a). (Whether you use the extended array approach or not is up to you.)

(c) Approximate g_σ by a Chebyshev polynomial of degree n with $m = n + 1$ Chebyshev nodes. Find the smallest value of n that achieves an error $r(n) < 0.01$. Do this for different values of σ. How does n depend on σ and why?

(d) Your job is now to approximate $v_{\alpha,\delta,\sigma}$ where $0 < \alpha < 1$, $0 \leq \delta \leq 1$, $\sigma > 0$. You can either approximate the function itself by a polynomial or define $\tilde{x} = \ln x$ and approximate $\tilde{v}_{\alpha,\delta,\sigma} = v_{\alpha,\delta,\sigma}(\exp \tilde{x})$ then compute $v_{\alpha,\delta,\sigma}(x) = \tilde{v}_{\alpha,\delta,\sigma}(\ln x)$ Which is better? How does the answer depend on the parameters?

3. (a) Write a Fortran subroutine that does one-dimensional linear interpolation and extrapolation. The subroutine should do the following: find $\tilde{y}$ the value of the underlying function, f, at the point $\tilde{x}$ given vectors x and y where if element i of x is x_i and element i of y is y_i then $y_i = f(x_i)$ and x and y are such that x is in ascending order.

   (b) For the three function forms in question 2, construct evenly-spaced grids of size n over their domains and generate the corresponding vectors of function values. Then use your linear interpolation subroutine to generate a piece-wise linear approximation for each function over a finer grid, i.e., a grid of size $m > n$.

   (c) Do the same, for the second two functions, with an initial grid that is evenly-spaced in the natural logarithm of x. Compare your results to the ones of part b. Which choice is better? Does it depend on your choice of parameter values?

4. (a) Use IMSL to create a one-dimensional cubic spline approximation to the three functions from question 2.

   (b) Compare your results to those you got from the piece-wise linear approximation you did in question 3 and the Chebyshev polynomial approximation you did in question 2. Which method worked best for each function? How does this answer depend on your choices of parameter values, grids, and/or number of nodes?

5. (a) Write a Fortran subroutine that finds the root of a one-dimensional nonlinear equation using Newton's Method. The method should compute and use a numerical approximation of the derivative.

   (b) Consider a representative agent economy where preferences are given by

$$U(c, n) = \frac{\left(c^\alpha (1-n)^{1-\alpha}\right)^{1-\sigma}}{1-\sigma},$$

and the technology is

$$y = zk^{\theta}n^{1-\theta}.$$

Capital, k, depreciates at rate δ. In addition to the standard budget constraint the agent faces a nonnegativity constraint on consumption and $0 \leq n \leq 1$. In the following, use $\beta = 0.98$, $\delta = 0.1$, $\theta = 0.36$, $\alpha = 0.6$ and $\sigma = 1.5$. Set z so that y is normalized to 1 in the steady state. Construct an evenly-spaced grid for capital with a lower bound, $k_{min} = 0.75k_{ss}$ and an upper bound $k_{max} = 1.25k_{ss}$, where k_{ss} is the steady state level of capital. For each possible combination of k and k' taken from the capital grid, compute the optimal labor supply, n, by solving the first-order condition for n using your nonlinear equation solver. Impose the constraint by setting values of n outside $[0, 1]$ to their appropriate values. Note: No need to go any further.