Economics 613/614

Fall 2007

**Assignment 2**

1. Consider a representative agent economy where preferences are given by

$$U(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma},$$

and the technology is

$$y = zk^{\alpha}.$$

Capital, $k$, depreciates at rate $\delta$. In the following, use $\beta = 0.98$, $\delta = 0.1$, $\alpha = 0.36$, and $\sigma = 2.0$. Consider $z$ as a parameter and include an equation to normalize $y$ to $1$ in the steady state.

(a) Using MATLAB, solve the Bellman equation using value function iterations. Use matrix operations to compute the updated value function at each iteration.

(b) Using MATLAB, recompute the value function using "for-loops" only instead of matrix operations. Compare the time needed by this algorithm to the time needed by the previous one. Which one takes longer? How does your answer depend on the number of points in your capital grid and your convergence criteria? Does it change when you change the calibration?

(c) Use the fact that the value function is concave and the policy function monotonic to improve your algorithm in part b . Again compare the computing times to the previous two cases and for different grid sizes, convergence criteria, and calibrations.

(d) Incorporate Howard's improvement algorithm into each of the programs from parts a, b, and c. Again compare the computing times.

(e) Solve the model again using value function iterations with FORTRAN and "do-loops". Compare the computing times to the MATLAB versions.

(f) Use the fact that the value function is concave and the policy function monotonic to improve your "do-loops" algorithm. Again compare the computing times.

(g) Incorporate Howard's improvement algorithm into your FORTRAN program. Again compare the computing times.

(h) Create a table summarizing your findings on computing time. Which approach is the most efficient? Does your answer depend on the grid size, the convergence criteria, and/or the calibration?

2. Consider a representative agent economy where preferences are given by

$$U(c,n) = \frac{\left(c^{\theta}(1-n)^{1-\theta}\right)^{1-\sigma}}{1-\sigma},$$

and the technology is

$$y = zk^{\alpha}n^{1-\alpha}.$$

Capital, $k$, depreciates at rate $\delta$. In addition to the standard budget constraint the agent faces a nonnegativity constraint on consumption and $0 \leq n \leq 1$. In the following, use $\beta = 0.98$, $\delta = 0.1$, $\theta = 0.6$, $\alpha = 0.36$ and $\sigma = 2.0$. Set $z$ so that $y$ is normalized to $1$ in the steady state.

(a) Solve the model using value function iterations with FORTRAN. Based on your analysis from question 1 you should decide whether or not to take advantage of concavity and monotonicity, and also whether or not to use Howard's. Note: You can use your code from homework 1 question 5 to obtain the optimal labor choice for each point on the $(k, k')$ grid.

(b) Modify your code from part a so that linear interpolation on the value function is used. You will need a routine that finds the maximum of a univariate function. Use the golden section search subroutine you wrote for homework 1 question 1.

(c) Compare the accuracy and efficiency of the two methods you implemented in parts a and b. Include in your write-up graphs of the policy and value functions.