THE UNIVERSITY OF WESTERN ONTARIO
LONDON ONTARIO

Paul Klein
Office: SSC 4028
Phone: 661-2111 ext. 85227
Email: paul.klein@uwo.ca
URL: www.ssc.uwo.ca/economics/faculty/klein/

Economics 613/614

Advanced Macroeconomics I & 2

Fall 2007

Homework 4

This homework is about solving the model in Huggett (93) and Huggett (97).

Let log individual earnings follow an AR(1) process with mean 0, autocorrelation ρ and one-step-ahead prediction error variance σ_ε^2. Find a source for a reasonable choice of ρ and σ_ε^2 and compute a finite state Markov chain approximation to this process. (One possible source is [**?**].) Denote the state space by $E = \{e_1, e_2, \ldots, e_m\}$ and the probability transition matrix by P.

1. Consider an environment where a continuum of agents each have an idiosyncratic earnings process e_t described by a finite-state Markov chain in discrete time with state space E and PTM P. Thanks to the law of

large numbers, there is no aggregate uncertainty. Preferences are represented by

$$E\left[\sum_{t=0}^{\infty}\beta^t \frac{c_t^{1-\sigma}-1}{1-\sigma}\right]$$

and the constraints are

$$c_t + qb_{t+1} = b_t + e_t$$

and

$$b_{t+1} \geq \underline{b}$$

and $b_0 \geq \underline{b}$. Set the parameters β, σ and $\underline{b} < 0$ as you like. Notice that the only asset traded is a risk-free bond. These bonds are in zero net supply. Your job is to find the equilibrium bond price $q < \beta$ as described below.

(a) Define a function f that for each value of the bond price q produces the aggregate demand for bonds. Do this in the following steps (or otherwise).

i. Solve for the individual savings functions using the method of endogenous grids and piecewise linear interpolation. Call these savings functions $g_i(b)$ for $i = 1, 2, \ldots, m$ and define $\bar{b}$ via $g_m(\bar{b}) = \bar{b}$. The domain of each savings function is now $[\underline{b}, \bar{b}]$.

ii. Create a (fine) new grid on $[\underline{b}, \bar{b}]$ for the purpose of creating a finite-state Markov chain approximation of the evolution of the state. Call this grid $\mathbf{b} = \{b_1, b_2, \ldots, b_n\}$.

iii. Use the savings functions $g_i(b)$ to define vectors of integers $\vec{g}_i$ via the following recipe. $\vec{g}_i(j)$ is the number k such that $b_k \leq g_i(b_j) < b_{k+1}$.

iv. Organize the state space as follows.

$$\mathcal{X} = \{(b_1, e_1), (b_1, e_2), \ldots, (b_n, e_{m-1}), (b_n, e_m)\} = \{x_1, x_2, \ldots, x_{nm}\}.$$

Given this definition, we can define the functions $i(k)$ and $j(k)$ via

$$i(k) = \mathrm{floor}((k-1)/m) + 1$$

and

$$j(k) = k - m\,\mathrm{floor}((k-1)/m)$$

where $\mathrm{floor}(x)$ is the greatest integer n such that $n \leq x$. With these definitions in place, define a PTM Q on $\mathcal{X}$ via

$$Q_{s,t} = \begin{cases} 0 & \text{if } i(s) = \vec{g}_{j(t)}(i(t)) \\ P_{j(s),j(t)} & \text{otherwise} \end{cases}$$

v. Find the stationary probability distribution associated with Q. Hence find aggregate asset holdings.

(b) Solve for the value of q such that $f(q) = 0$.

2. Now let there be capital (keeping the model otherwise unchanged). Let the aggregate resource constraint be

$$K_{t+1} + C_t = zK_t^{\theta} L^{1-\theta} + (1-\delta)K_t$$

where z is chosen so that aggregate output in an economy without idiosyncratic shocks is equal to 1 and L is aggregate labour supply.[1] Set $0 < \delta < 1$, $0 < \theta < 1$ as you like. This time, write the budget constraint via

$$c_t + a_{t+1} = ra_t + we_t$$

(this is just a formality and not a substantive change). Assume that the borrowing constraint is

$$a_{t+1} \geq 0.$$

[1] L is the sum or average of all the individual time endowments e_t^i. It doesn't depend on time. It is a good idea to normalize things so that $L = 1$.

(a) Define a function f that for each value of aggregate capital K gives aggregate savings by going through the following steps (or otherwise).

   i. Fix K, hence find r and w by profit maximization.

   ii. Given r and w, find the stationary distribution of assets.

   iii. Given the stationary distribution of assets, find aggregate asset holdings.

(b) Find the K such that $f(K) = K$.

(c) Is the equilibrium value of K greater than or less than it would be in an economy without idiosyncratic uncertainty?

References