ASSIGNMENT 5
ECO 613/614
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1. Consider the following perfect foresight life-cycle model. Time is discrete. In each period a new generation is born into the economy and the eldest generation dies. There is no population growth so each generation is of equal measure and the total measure of households in the economy is constant over time and normalized to $1$. Households live for $T$ periods, the first $R$ of which they spend working and the last $T-R$ of which they are retired. The date of retirement $R$ is exogenous. Households are endowed with one unit of time each period. Working households choose how to allocate their time between work and leisure. Retired households spend all their time on leisure. Households receive income from capital and labor and allocate it between consumption and savings.

**Households' Problem.** The problem of an age 1 household born at date $t$ is to choose sequences of consumption, $\{c^s_{t+s-1}\}_{s=1}^T$, labor, $\{n^s_{t+s-1}\}_{s=1}^R$, and capital holdings, $\{k^s_{t+s-1}\}_{s=2}^{T+1}$, to maximize life-time utility,

$$\sum_{s=1}^{T} \beta^{s-1} u(c^s_{t+s-1}, 1-n^s_{t+s-1}),$$

subject to

$$(1+\tau^c_{t+s-1})c^s_{t+s-1} + k^{s+1}_{t+s} = k^s_{t+s-1} + (1-\tau^k_{t+s-1})r_{t+s-1}k^s_{t+s-1} + \\ (1-\tau^n_{t+s-1})w_{t+s-1}n^s_{t+s-1}, \quad \text{for } s=1,\dots,R,$$

$$(1+\tau^c_{t+s-1})c^s_{t+s-1} + k^{s+1}_{t+s} = k^s_{t+s-1} + (1-\tau^k_{t+s-1})r_{t+s-1}k^s_{t+s-1} + b, \quad \text{for } s=R+1,\dots,T$$

$k^1_t = k^{T+1}_{t+T} = 0$ and $n^s_t = 0$ for $s = R+1,\dots,T$. Here, $w_{t+s-1}$ is the wage rate in period $t+s-1$ when the household is age $s$ and $r_{t+s-1}$ is the net interest rate in period $t+s-1$. Labor earnings in period $t+s-1$ are taxed at rate $\tau^n_{t+s-1}$, capital income is taxed at rate $\tau^k_{t+s-1}$ and consumption is taxed at rate $\tau^c_{t+s-1}$. Each period of retirement agents receive an exogenously-given public pension $b$. The period utility function is

$$u(c,n) = \frac{(c^\theta n^{1-\theta})^{1-\sigma}}{1-\sigma}.$$

**Firms' Problem.** A measure one of firms combine capital $K_t$ and labor $N_t$ in period $t$ to produce output $Y_t$ using the standard Cobb-Douglas technology

$$Y_t = F(K_t, N_t) = K_t^{\alpha} N_t^{1-\alpha}.$$

Capital is rented from agents at rate $r_t$ and depreciates at rate $\delta$ and labor is rented from agents at rate $w_t$. Firm profit maximization yields

$$w_t = (1-\alpha) K_t^{\alpha} N_t^{1-\alpha},$$

$$r_t = \alpha K_t^{\alpha-1} N_t^{1-\alpha} - \delta.$$

**Government.** The government chooses the tax rates on labor earnings, capital income, and consumption at each date $t$, $\{\tau_t^n, \tau_t^k, \tau_t^c\}$ these choices must be such that the government budget is balanced, i.e., so that total taxes collected is equal total expenditure on old-age pensions plus government consumption at each date $t$,

$$\tau_t^n w_t N_t + \tau_t^k r_t K_t + \tau_t^c C_t = \frac{T-R}{T} b$$

and the choice of taxes is consistent with the given government policy for period $t$.

**Equilibrium.** An equilibrium for a given government policy and initial distribution of capital $\{k_0^s\}_{s=1}^T$ consists of sequences of lifetime profiles of consumption, labor supply, and capital holdings, $\{\{c_{t+s-1}^s\}_{s=1}^T, \{n_{t+s-1}^s\}_{s=1}^R, \{k_{t+s-1}^s\}_{s=1}^T\}$, prices of labor and capital $\{w_t, r_t\}$, and taxes $\{\tau_t^n, \tau_t^k, \tau_t^c\}$ such that

1. The allocations solve the agents' problems given prices and taxes.
2. Factors are paid their marginal product.
3. Individual and aggregate behavior are consistent, i.e.,

$$N_t = \frac{1}{T} \sum_{s=1}^{R} n_t^s, \quad K_t = \frac{1}{T} \sum_{s=1}^{T} k_t^s, \quad C_t = \frac{1}{T} \sum_{s=1}^{T} c_t^s.$$

4. Taxes are such that the government budget is balanced and the government policy is met.
5. The resource constraint holds, i.e.,

$$Y_t = C_t + K_{t+1} - (1-\delta) K_t.$$

(a) Compute the steady state of this economy for $T = 60$ and $R = 40$ and reasonable values for the other parameters under the assumption that there is no transfer to retired individuals, i.e., $b = 0$ and all tax rates are $0$. Check that the resource constraint holds to verify that you have successfully computed the steady state.

(b) Now find the steady state allocations, prices, and taxes for the same calibration but with $b$ set to $50\%$ of the average period income of workers in the initial steady state and 3 different tax scenarios:

   i. the transfer is financed by labor income taxes alone,

   ii. the transfer is financed by capital income taxes alone,

   iii. the transfer is financed by consumption taxes alone.

   Again you should verify that the resource constraint holds in each case.

(c) Compare welfare in the four different steady states.

(d) Suppose that the economy is in the steady state of part a and all of a sudden at date $1$ the government announces that starting in the current period their will be a tax-transfer system that gives retired individuals the value $b$ you found in part b from now on. Compute the transition to from the initial steady state to the new steady state for under the three different financing schemes given in part b. Again verify that in each period along the transition the resource constraint holds.

(e) Who wins and loses along the transition? How does your answer depend on the financing scheme chosen by the government?