THE UNIVERSITY OF WESTERN ONTARIO
LONDON ONTARIO

Paul Klein
Office: SSC 4028
Phone: 661-2111 ext. 85227
Email: paul.klein@uwo.ca
URL: www.ssc.uwo.ca/economics/faculty/klein/

Economics 613/614

Advanced Macroeconomics I & 2

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Homework 6

This homework is about solving the models in [Kocherlakota, 1996] and [Krueger and Perri, 2005].

1. Consider a version of the environment in [Kocherlakota, 1996]. There are two agents, $i = 1$ and $i = 2$. The aggregate endowment is equal to 1 in each period. The state of the economy $\theta_t \in \{\ell, h\}$ follows a two–state symmetric Markov chain. In state h, the endowment of agent 1 is $\omega_h \geq 0.5$ and in state 2 it is $\omega_\ell = 1 - \omega_h$. The probability of staying in the current state is $0 \leq \gamma < 1$. The subjective discount factor is $0 \leq \beta < 1$ and the period utility function is given by

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}$$

if $\sigma > 0$ and $\sigma \neq 1$ and

$$u(c) = \ln c$$

if $\sigma = 1$.

(a) Verify that, in a long run efficient allocation featuring some but not perfect risk sharing, consumption by agent 1 in state h is $0.5 < c_h < \omega_h$ and consumption by agent 1 in state ℓ is $c_\ell = 1 - c_h$ and

$$\frac{u(\omega_h) - u(c_h)}{u(c_\ell) - u(\omega_\ell)} = \frac{\beta(1-\gamma)}{1-\beta\gamma}.$$

(b) Show that the degree of risk sharing $\omega_h - c_h$ is

i. increasing in β

ii. decreasing in γ

iii. increasing in σ.

(c) Explain intuitively the results in (b).

2. Modify the environment so that $\theta_t \in \{\ell, m, h\}$ and let the endowment of agent 1 (and also of agent 2) in state m be 0.5. Let the probability of staying in the current state be $0 \leq \gamma < 1$ for all states. The probability of going to any other particular state is $(1-\gamma)/2$. Suppose the endowment of agent 1 in states h and ℓ are ω_h and $\omega_\ell = 1 - \omega_h$, respectively.

(a) Describe the qualitative properties of a long–run efficient allocation with some but not perfect risk–sharing.

(b) Derive explicit expressions for the autocorrelation of the endowment and of individual consumption. Verify that the latter is non–negative and strictly greater than the former.

(c) Compute a long–run efficient allocation with some but not perfect risk–sharing. Calibrate as you like.

3. Consider an environment with a continuum of individuals of total measure one. Let individual endowments be independent of each other so that the aggregate endowment is constant and equal to one. Each endowment process follows a Markov chain with two states, y_L and y_H. Calibrate as you like and compute a stationary general equilibrium with some but not perfect risk sharing.

References

[Kocherlakota, 1996] Kocherlakota, N. R. (1996). Implications of efficient risk sharing without commitment. *Review of Economic Studies*, 63(4):595–609.

[Krueger and Perri, 2005] Krueger, D. and Perri, F. (2005). Public versus private risk sharing. Manuscript.